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Investigation of a partitioned cavity silencer using a woven metal screen as acoustic liner and sound absorber

April 10th, 2019

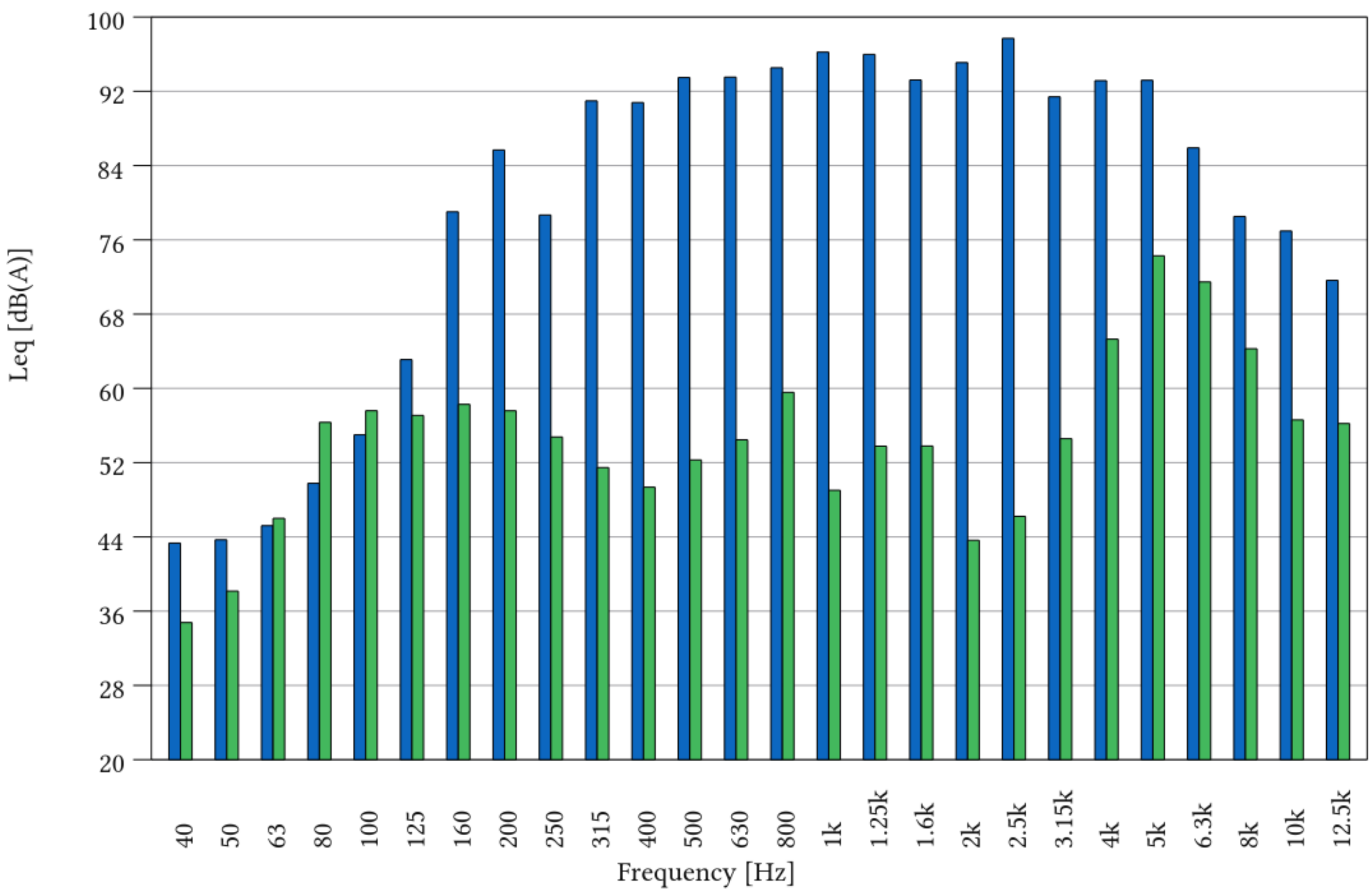
Exhaust Engineering

Overview

- Demo
- Working principle
- Silencer model
  - Material measurements
  - 1D Model derivation
- Comparisons with FEM (transmission loss)
- Comparison with measurements (insertion loss)
- Conclusions
- Questions

Demo...

Demo...



Overall A-weighted level difference: 28 dB

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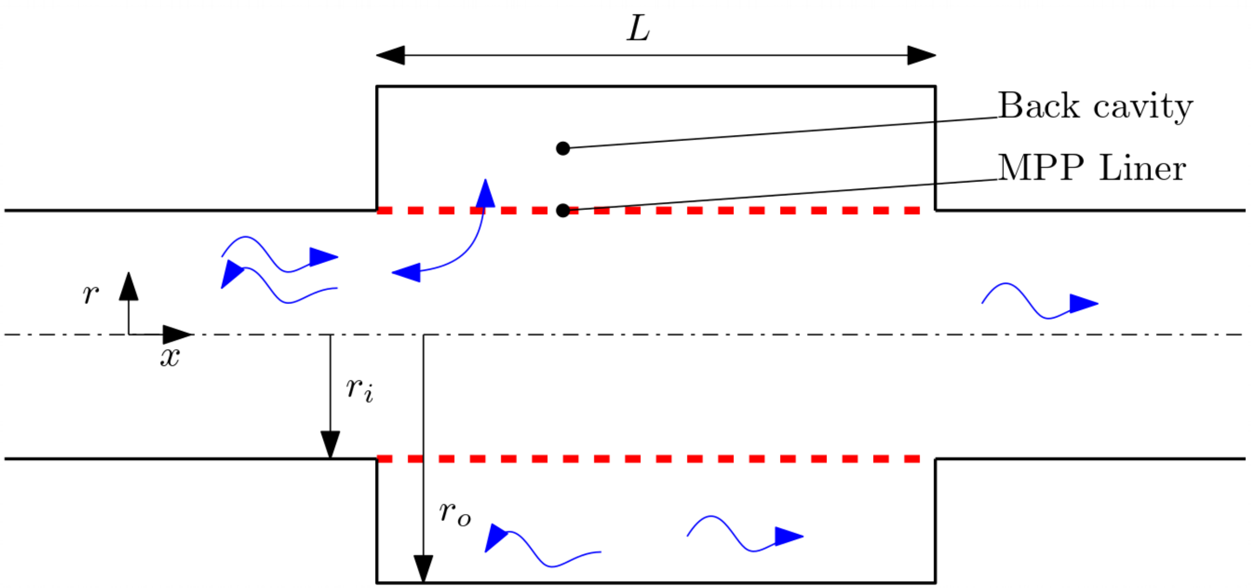
Working principle

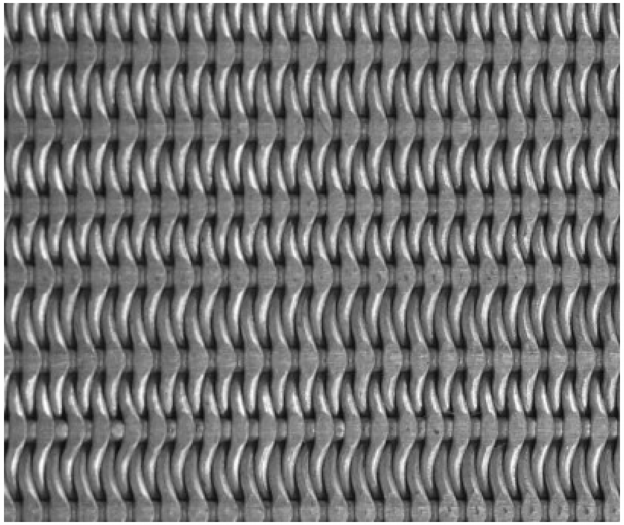
Working principle



- Liner: pores which are small compared to viscothermal diffusion layer thickness $\delta = \sqrt{\frac{2\mu}{\rho_0 \omega}}$
  - Viscothermal dissipation in the liner material
  - Back cavity function is allowing

Working principle

- Liner: woven stainless steel
  - Porosity ~ 1%
  - Pore size ~ 0,07 mm
  - Thickness ~ 1 mm



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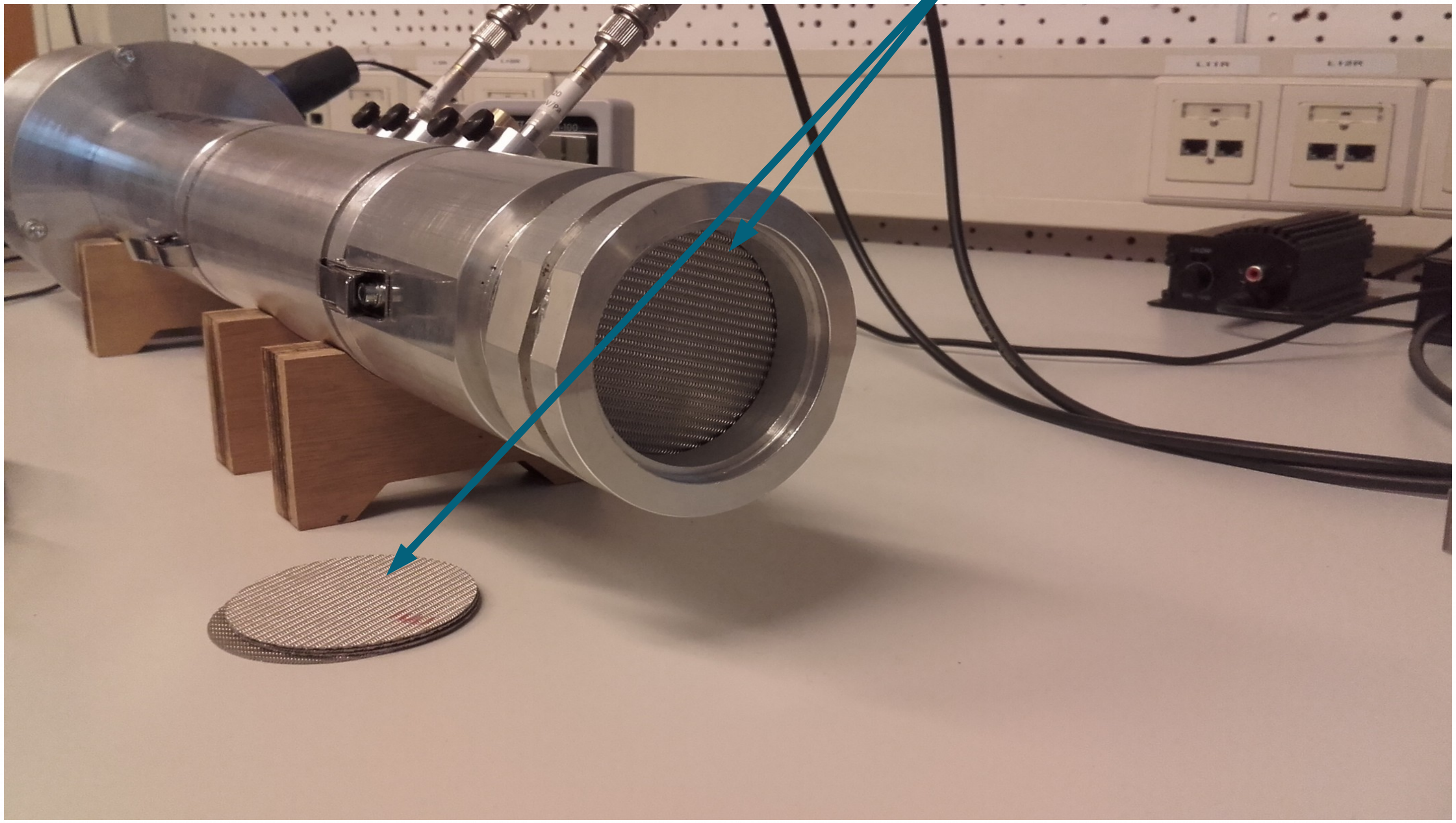
Silencer model
Material measurements

Material measurements

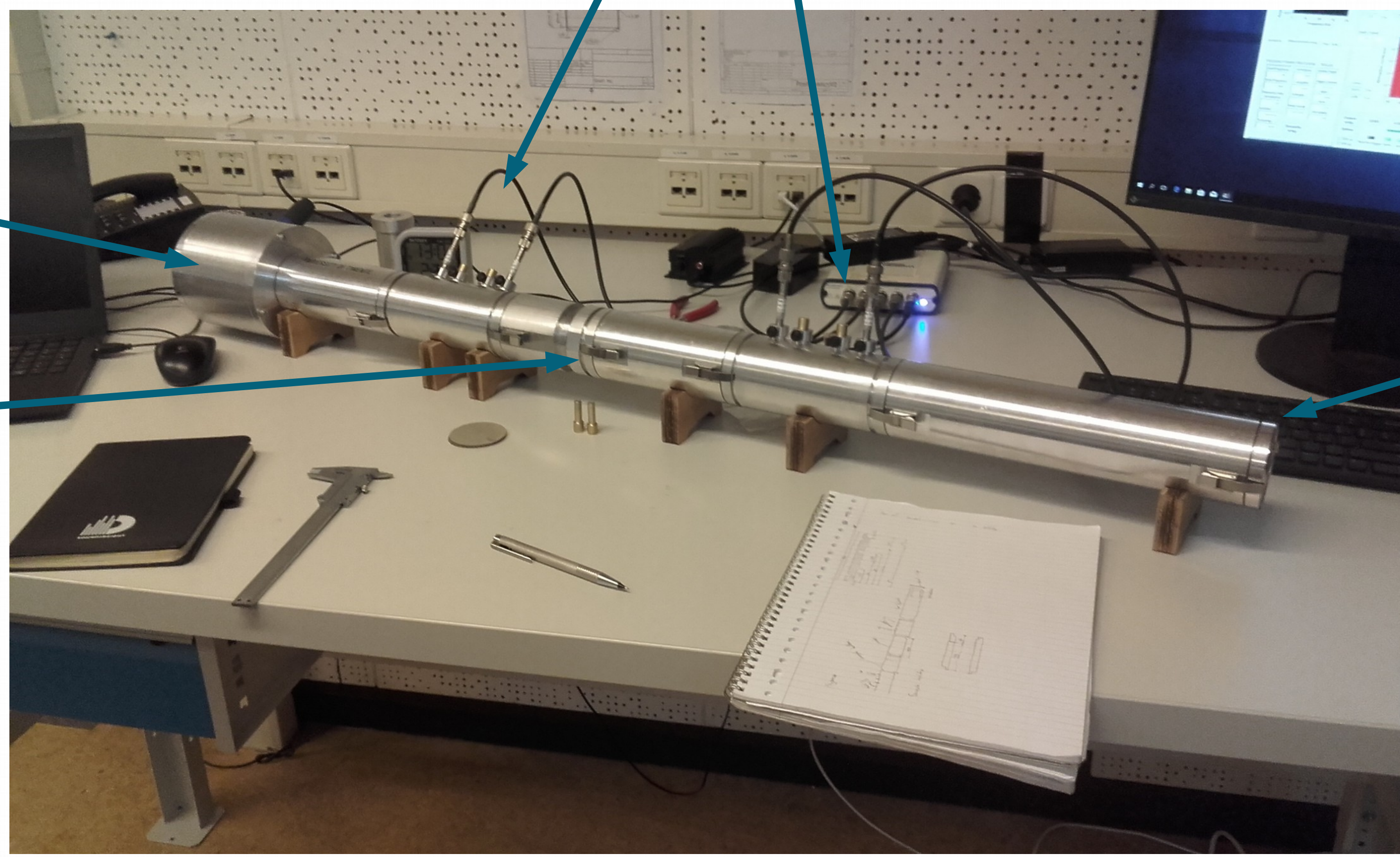
- Impedance tube UT

Sample

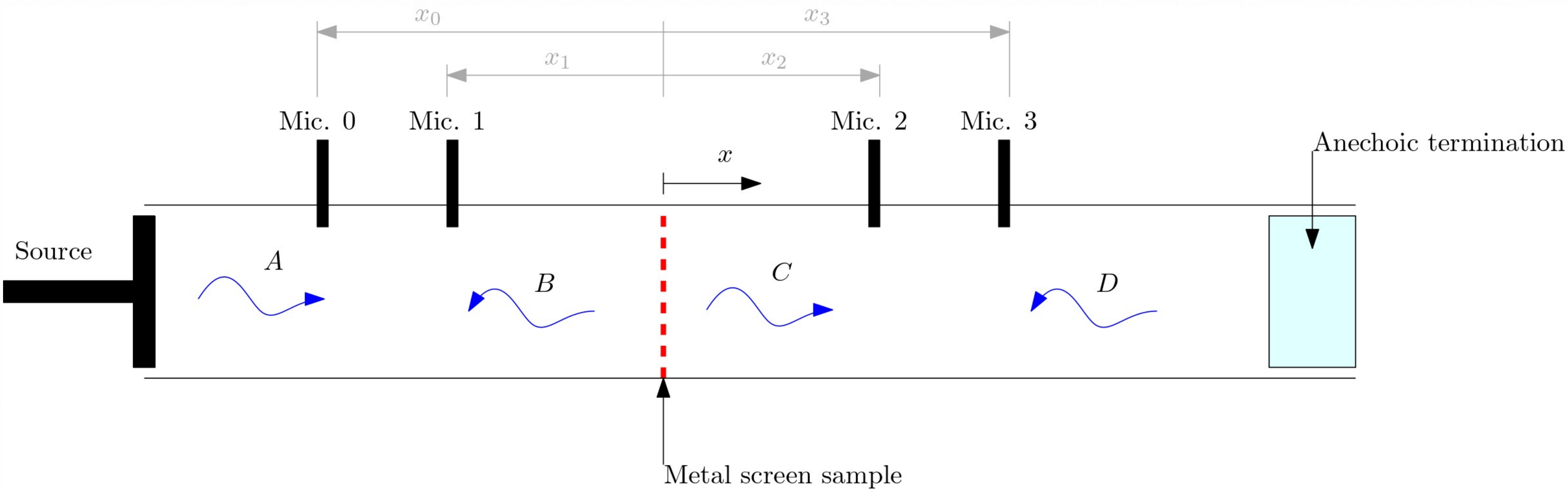


Material measurements

- Impedance tube



Material measurements



$$p_0 = A\exp(-ikx_0) + B\exp(ikx_0)$$
$$p_1 = A\exp(-ikx_1) + B\exp(ikx_1)$$
$$p_2 = C\exp(-ikx_2) + D\exp(ikx_2)$$
$$p_3 = C\exp(-ikx_3) + D\exp(ikx_3)$$

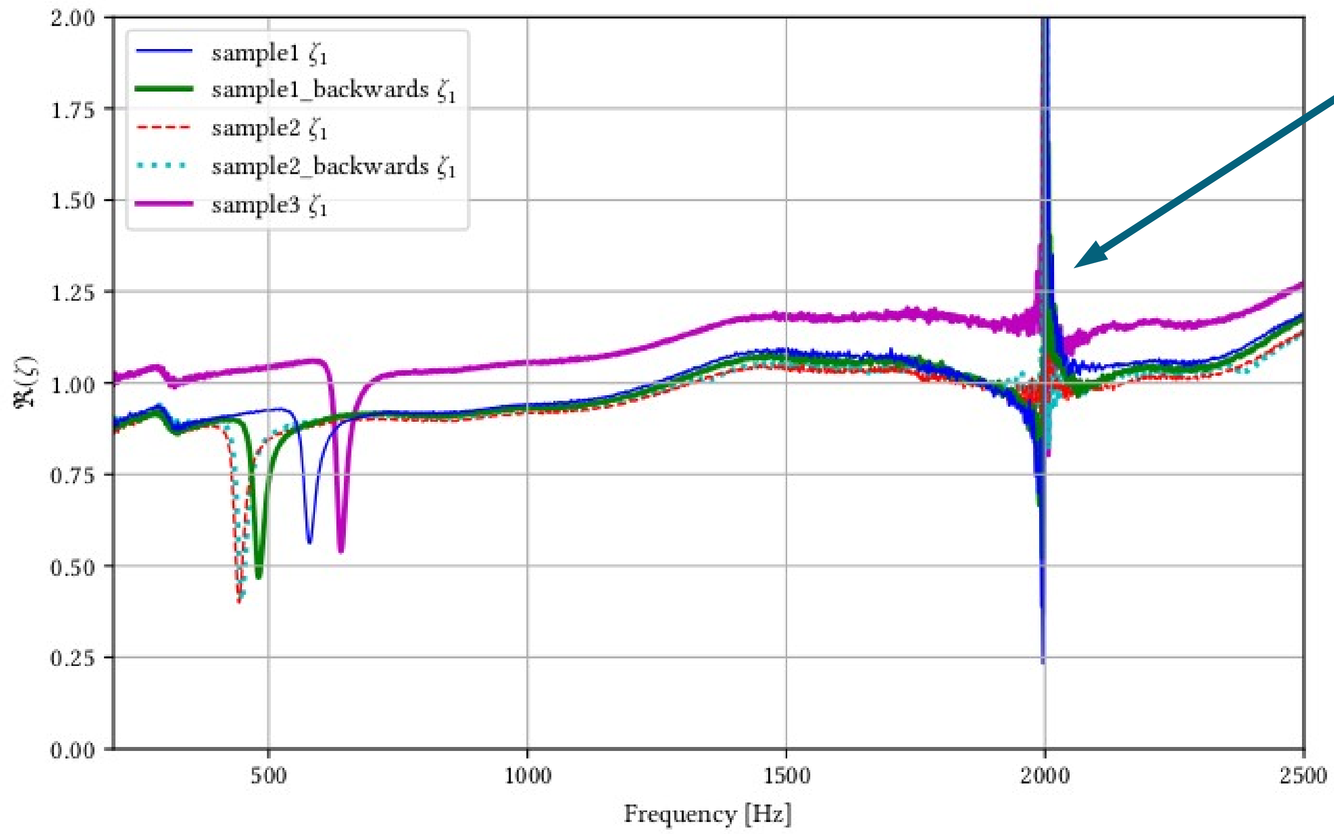
4 equations, 4 unknowns

$$\Delta p = (C + D) - (A + B)$$
$$u = \frac{1}{z_0}(A - B)$$

$$\zeta(\omega) = -\frac{1}{z_0}\frac{\Delta p(\omega)}{u(\omega)} = \frac{(A + B) - (C + D)}{A - B}$$

Material measurements

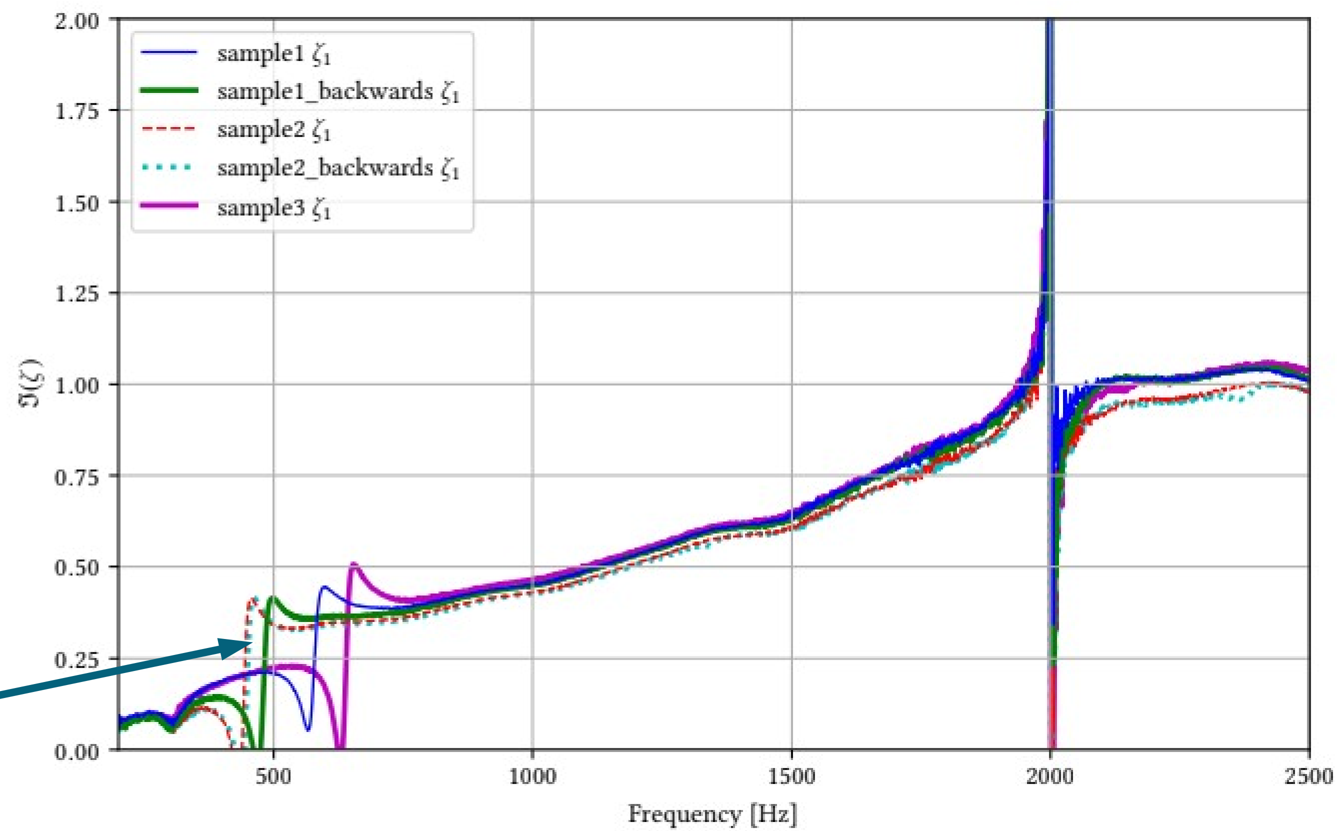
- Real part of normalized impedance



Mic distance = lambda/2

Material measurements

- Imaginary part of normalized impedance



First bending mode of plate? (sensitive to clamping force)

Material model

- Linear empirical fit:

$$\zeta(\omega) = 1 + i\,\frac{\omega/2\pi}{2000}$$

- More advanced models:
  - Johnson-Champoux-Allard
  - Micro-perforated plates

- End result for silencer not really sensitive to variations in zeta.

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Silencer model
1D model

1D Silencer model

- Assumptions
  - Linear, isentropic acoustics in cavity and main passage
  - Cavity transverse size is small compared to the wavelength:
  - Axial propagation of waves in the main passage and back cavity is allowed $r_o \ll \lambda$
  - The liner wall thickness is small compared to the wavelength
    - Liner effect can be modeled as a lumped impedance jump
    - Velocity reacts locally to pressure difference across liner (not locally reacting liner impedance!)

1D Silencer model

- Continuity equation for the cavity:

$$\frac{i\omega S_c}{c_0^2} p_c + S_c \rho_0 \frac{\mathrm{d}u_c}{\mathrm{d}x} = \Pi \rho_0 u_r$$

- Continuity equation for the main channel:

$$\frac{i\omega S_i}{c_0^2} p_i + S_i \rho_0 \frac{\mathrm{d}u_i}{\mathrm{d}x} = -\Pi \rho_0 u_r$$

- Cavity – inner duct communication:

$$z_{\mathrm{liner}} u_r(x) = p_i(x) - p_c(x)$$

1D Silencer model

- Momentum equation for the cavity:

$$u_c = \frac{i}{kz_0}\frac{\mathrm{d}p_c}{\mathrm{d}x}$$

- Momentum equation for main channel:

$$u_i = \frac{i}{kz_0}\frac{\mathrm{d}p_i}{\mathrm{d}x}$$

1D Silencer model

- Combined:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + k_c^2\right) p_c = \left(k_c^2 - k^2\right) p_i$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + k_i^2\right) p_i = \left(k^2 - k_i^2\right) p_c$$

where

$$k_c^2 = k^2 - \frac{i\Pi k}{S_c \zeta_\mathrm{liner}}$$

$$k_i^2 = k^2 - \frac{i\Pi k}{S_i \zeta_\mathrm{liner}}$$

- Coupled set of ODE’s for pressure in back cavity and in main passage

1D Silencer model

- Solution procedure:
  - Ansatz for back cavity solution: $$p_c = \sum_{n=0}^{\infty} C_n \cos\left(\frac{n\pi}{L_c}x\right)$$
    - Substitution for p_c in ODE for p_i,
    - Substitution of result for p_i in terms of p_c back into ODE for p_i
  - Integrations along the length of the back cavity
    - Using orthogonality relations of the cosines with different spatial frequencies
  - Tedious....

1D Silencer model

- Solution:
  - Transfer matrix relation between pressure and velocity on one side of the liner, to the other side:

$$\begin{Bmatrix} p \\ z_0 u \end{Bmatrix}_0 = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} p \\ z_0 u \end{Bmatrix}_L$$

$$T_{11} = \frac{G_2 G_7 - G_1 G_8}{G_8 G_7 - G_5 G_8}$$

$$T_{12} = \frac{G_1 G_6 - G_2 G_5}{G_8 G_7 - G_5 G_8}$$

$$T_{21} = \frac{G_4 G_7 - G_3 G_8}{G_8 G_7 - G_5 G_8}$$

$$T_{22} = \frac{G_3 G_6 - G_4 G_5}{G_8 G_7 - G_5 G_8}$$

$$G_1 = 1 + \sum_{n=0}^{\infty} F_+ H_\alpha$$

$$G_2 = 1 + \sum_{n=0}^{\infty} F_- H_\alpha$$

$$G_3 = K_+ - \sum_{n=0}^{\infty} F_+ J_\beta$$

$$G_4 = -K_- - \sum_{n=0}^{\infty} F_- J_\beta$$

$$G_5 = e^{-ik_+L} + \sum_{n=0}^{\infty} F_+ H_\alpha (-1)^n$$

$$G_6 = e^{-ik_-L} + \sum_{n=0}^{\infty} F_- H_\alpha (-1)^n$$

$$G_7 = K_+ e^{-ik_+L} - \sum_{n=0}^{\infty} F_+ J_\beta (-1)^n$$

$$G_8 = -K_- e^{ik_-L} - \sum_{n=0}^{\infty} F_- J_\beta (-1)^n$$

$$K_+ = \frac{k_+}{k - k_+ M}$$

$$K_- = \frac{k_-}{k + k_- M}$$

$$H_\alpha = \frac{\left(\frac{k_a^2 - k^2}{1-M^2}\right)\left[\frac{k_a^2}{1-M^2} - \left(\frac{n\pi}{L}\right)^2 - \left(\frac{M^2}{1-M^2}\right)\left(\frac{k_a^2+k^2}{k^2}\right)\left(\frac{n\pi}{L}\right)^2\right]}{\left[\frac{k_a^2}{1-M^2} - \left(\frac{n\pi}{L}\right)^2\right]^2 - \left[\left(\frac{M}{1-M^2}\right)\left(\frac{k_a^2+k^2}{k^2}\right)\left(\frac{n\pi}{L}\right)^2\right]^2}$$

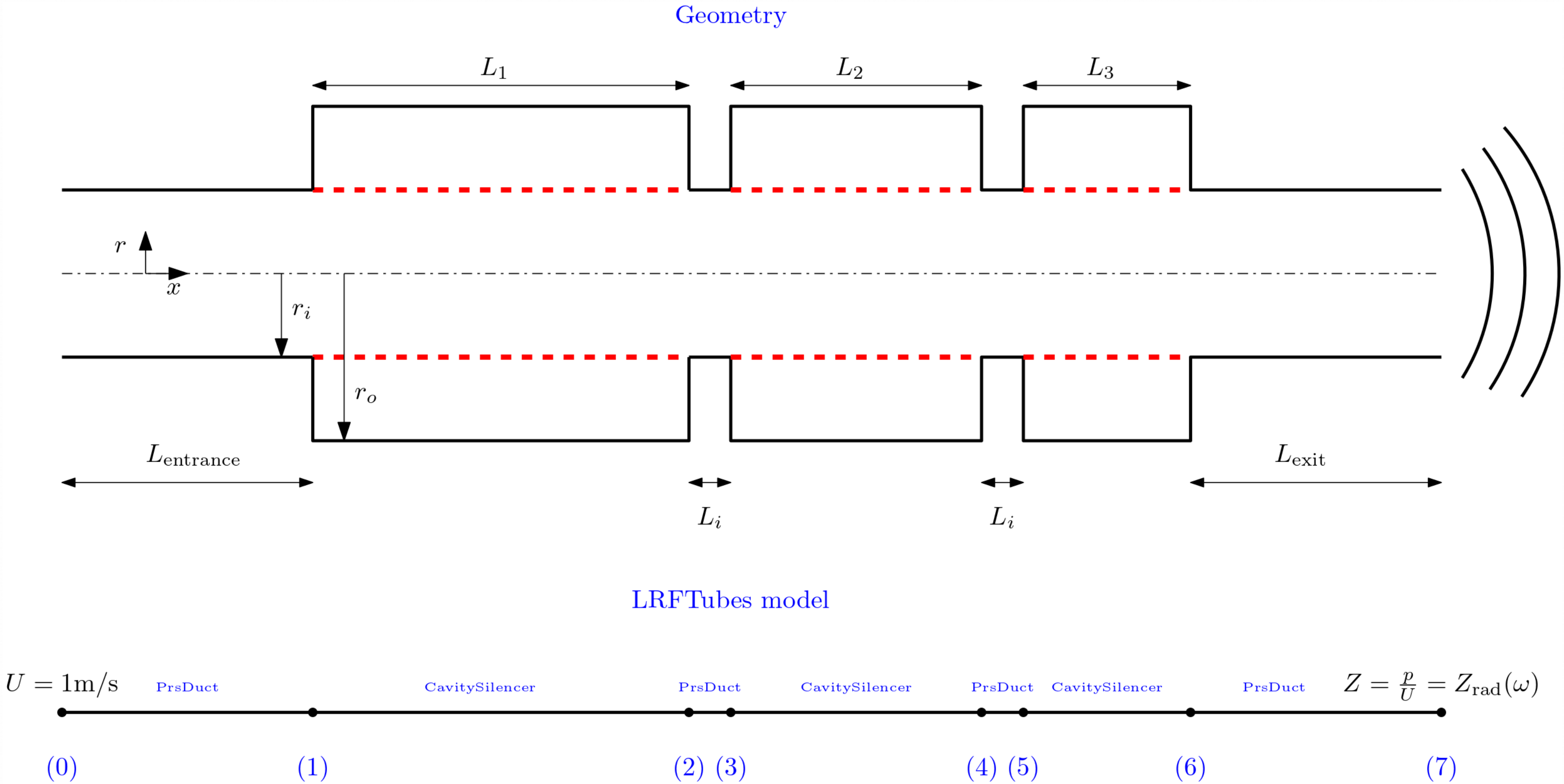
$$J_\beta = \left[\frac{\left(\frac{n\pi}{L}\right)}{M^2\left(\frac{n\pi}{L}\right)^2 - k^2}\right]\left(ikH_\beta^2 + \left(\frac{n\pi}{L}\right) M H_\alpha^2\right)$$

$$H_\beta = \frac{\frac{-iM}{1-M^2}\left(\frac{n\pi}{L}\right)\left(\frac{k_a^2 - k^2}{k}\right)\left[\frac{k^2}{1-M^2} + \left(\frac{n\pi}{L}\right)^2\right]}{\left[\frac{k_a^2}{1-M^2} - \left(\frac{n\pi}{L}\right)^2\right]^2 - \left[\left(\frac{M}{1-M^2}\right)\left(\frac{k_a^2+k^2}{k^2}\right)\left(\frac{n\pi}{L}\right)^2\right]^2}$$

$$F_\pm = \frac{\pm ik\left(k_b^2 - k^2\right)(2 - \delta_n)\left[(-1)^n e^{\mp ik_\pm L} - 1\right]}{L\left[k_\pm^2 - \left(\frac{n\pi}{L}\right)^2\right]\left[k_b^2 - \left(\frac{n\pi}{L}\right)^2\right] - H_\alpha\left(k_b^2 - k^2\right)}$$

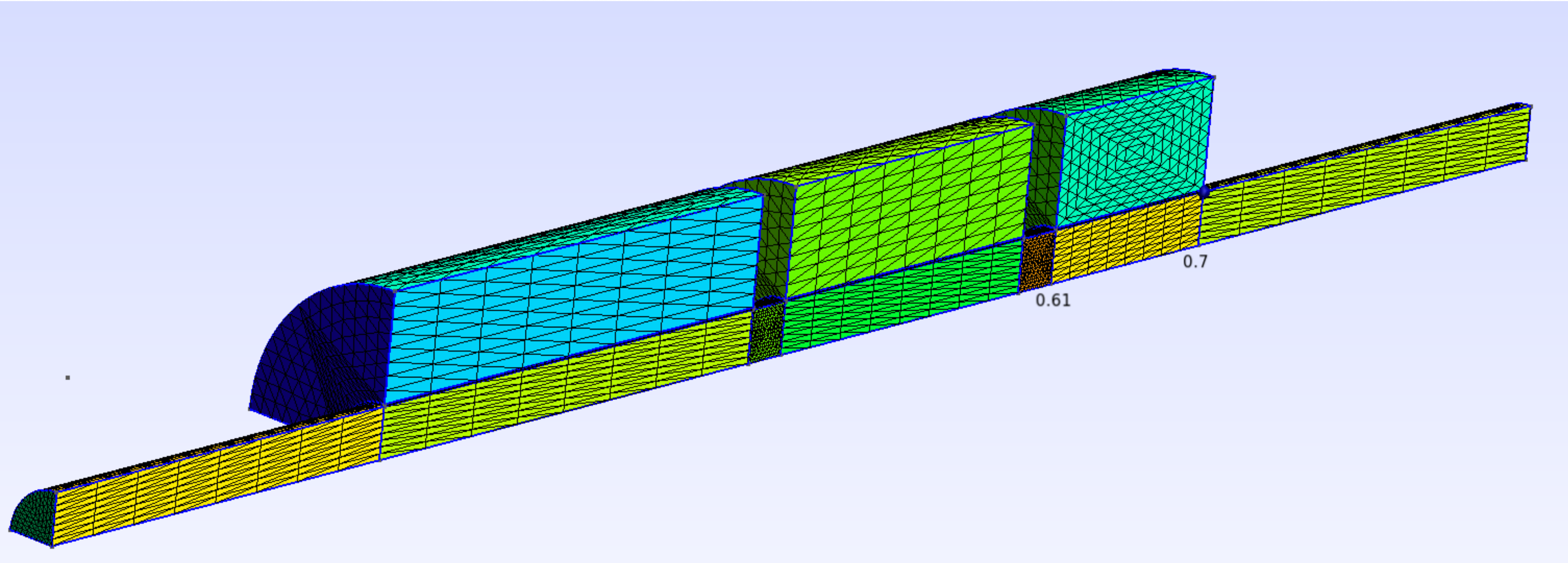
$$k_\pm = \frac{1}{1-M^2}\left(\sqrt{k_a^2 + M^2\left[\left(k_a^2 - k^2\right)/2k\right]^2} \mp M\left(\frac{k_a^2 + k^2}{2k}\right)^2\right)$$

Partitioned cavity silencer



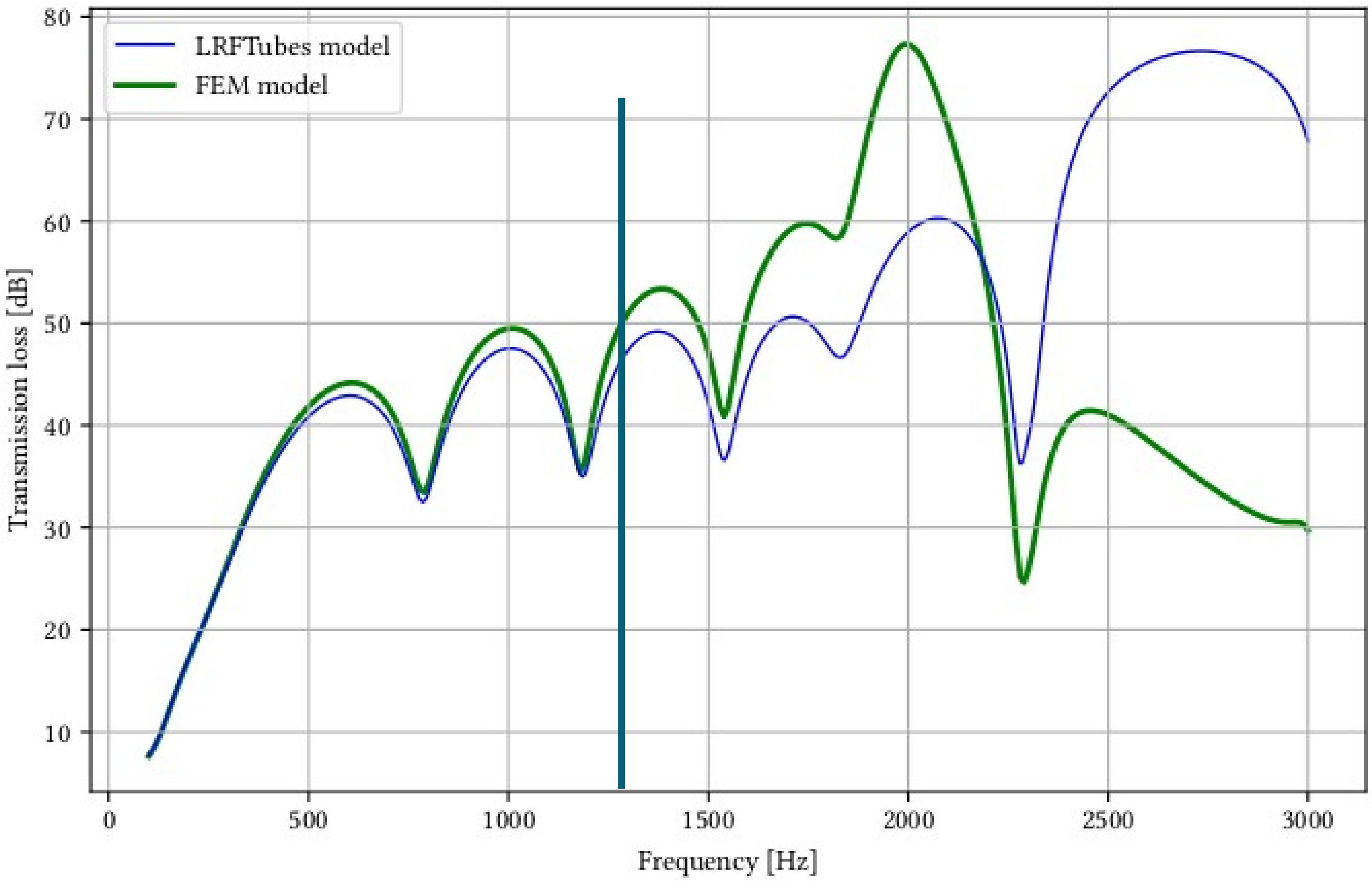
Transmission loss – comparison FEM

- ¼ th of the geometry (could be 2D axisymetric)
- Overly fine mesh

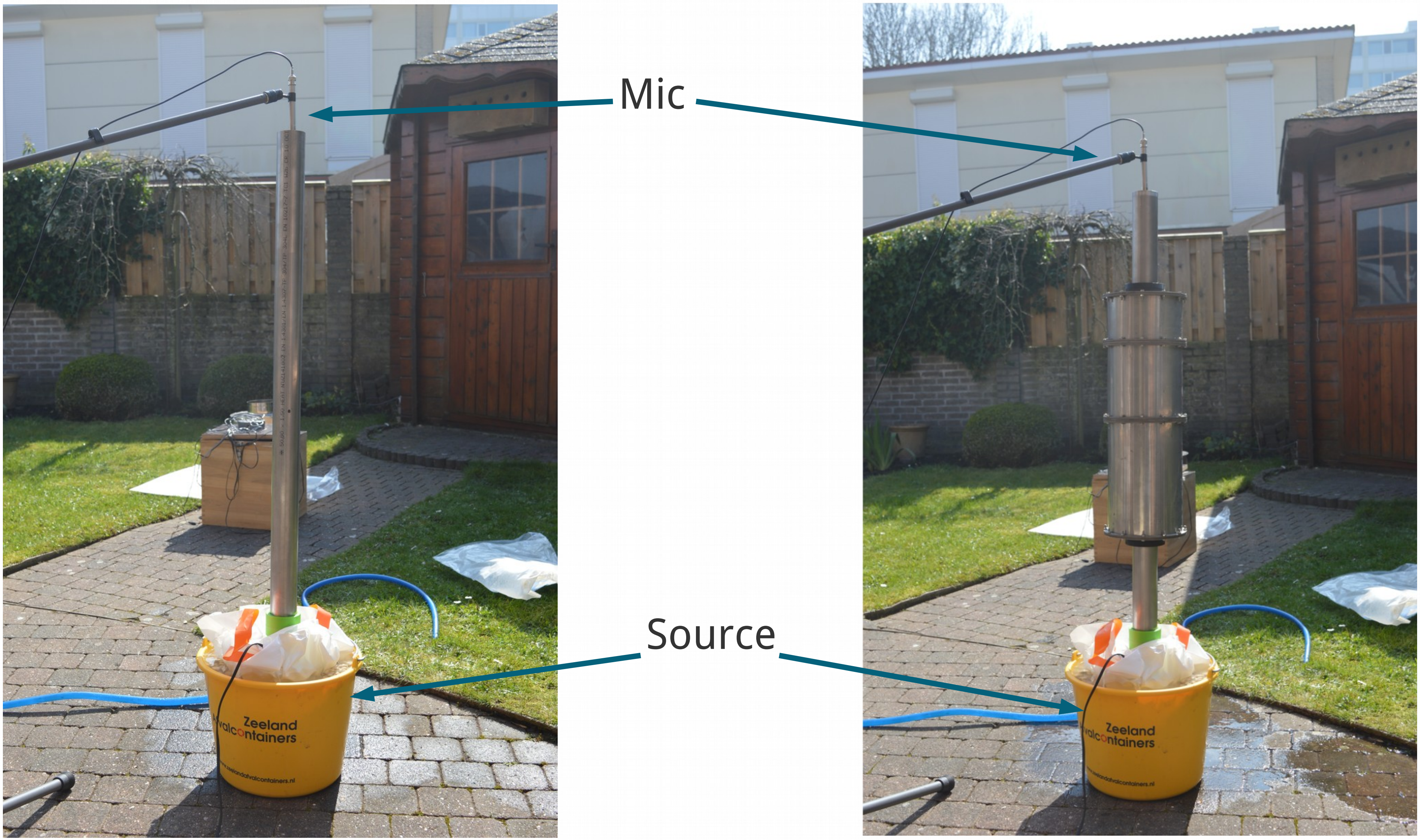


Transmission loss – comparison FEM

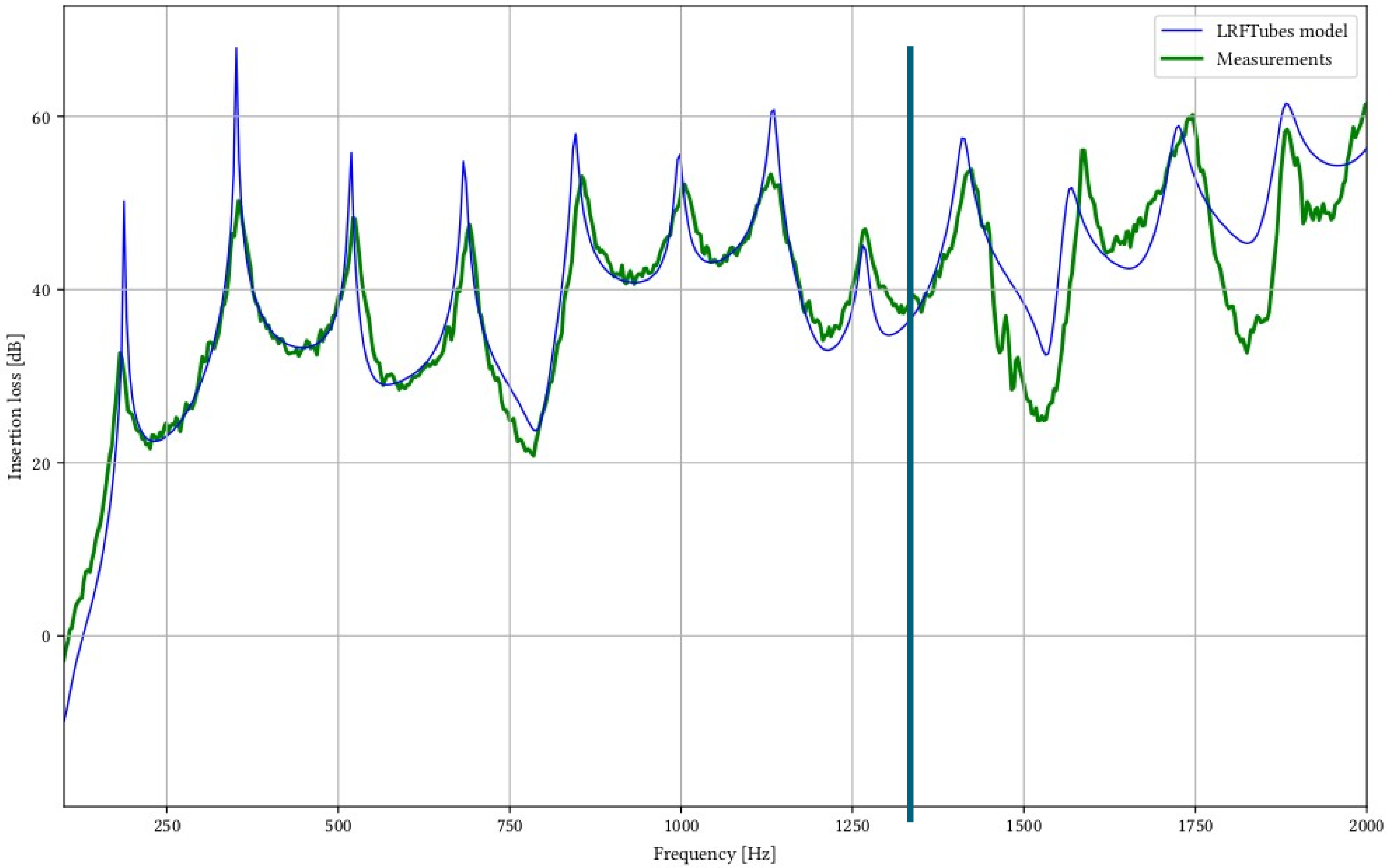
- Vertical line: cut-on frequency



Insertion loss measurements



Comparison of insertion loss



Conclusions

- Numerical model for a partitioned cavity silencer is implemented, based on the Sullivan-Crocker model
- Implementation is verified using a comparison of the transmission loss with FEM results
- The model is validated using experimental measurements

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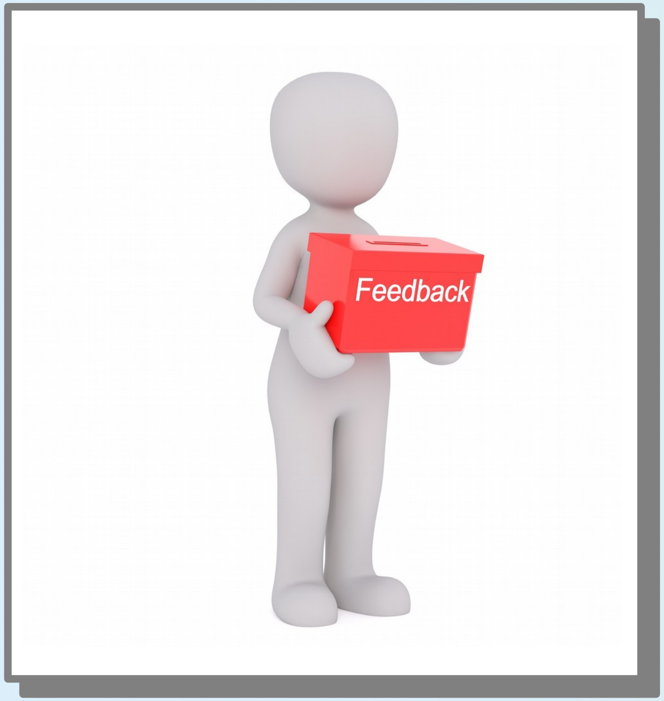
The end



References

- Sullivan, J. W., and Crocker, M. J. (1978). “**Analysis of concentric-tube resonators having unpartitioned cavities**,” The Journal of the Acoustical Society of America 64, 207–215.
- De Jong, J. A. (2015-2019). **LRFTubes - A Python code for computing 1D viscothermal acoustic waves in waveguides,** https://code.ascee.nl/ASCEE/lrftubes
- https://fenicsproject.org

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