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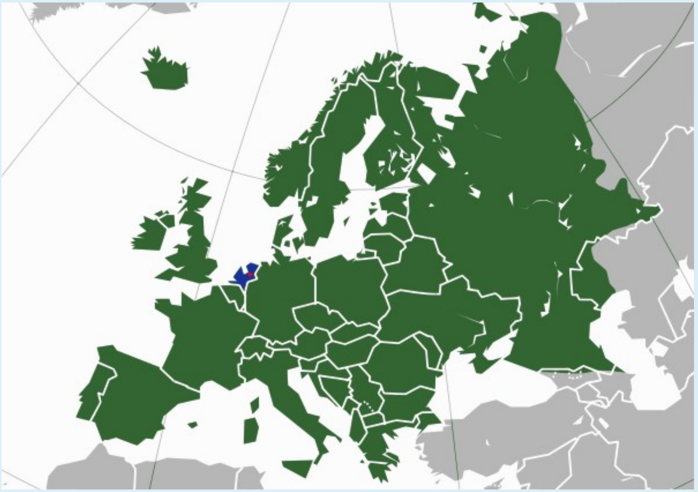

Parametric acoustic array

Anne de Jong

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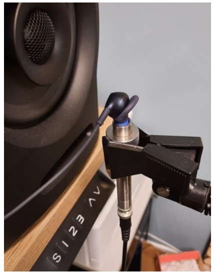

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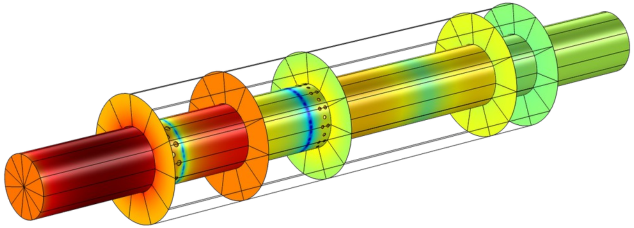
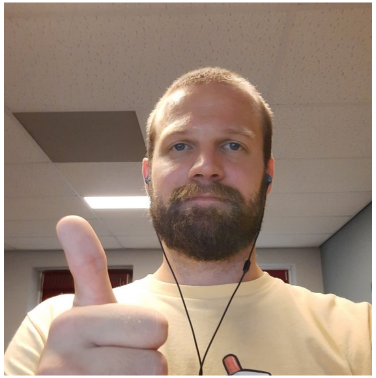
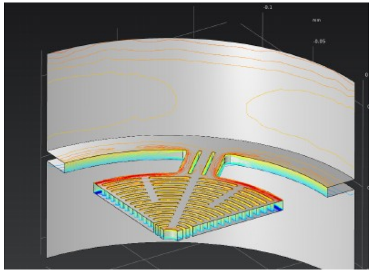




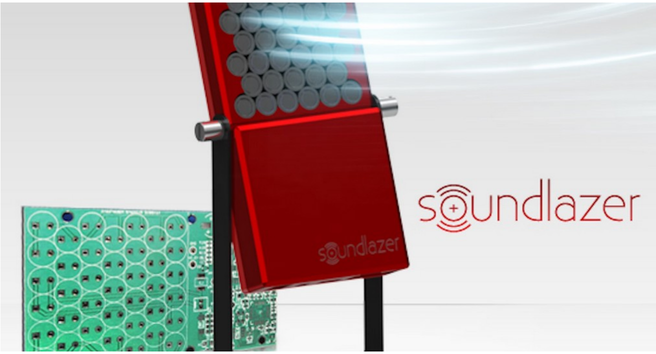


Introduction

- Motivation:
  - We have a small “constant displacement” source, that is relatively inefficient in radiating sound power.
  - We want to increase its sound power.
  - The source is able to radiate rather well in the ultrasonic range.
- Idea:
  - Send out high frequency, high amplitude ultrasound waves. Due to nonlinear intermodulation distortion, an audible (“difference frequency”) wave is generated.
  - The theory has been developed mainly in de “60s with underwater acoustics applications.
  - Main question: can it help to increase the (low frequency) acoustic output of a small source

Product examples

- Soundlazer / Kickstarter (movie)
- HoloSonics: AudioSpotLight
- Others… (related to military industry)



Audible waves

- Assumptions:
  - 1 cm^2, circular "baffled source"
  - X_{max} = a constant 3 µm
- Model: far-field radiation from a baffled piston.
- On-axis maximum SPL:

$$L_{\mathrm{p,max}} = 61\,\mathrm{dB} + 40\log_{10}\left(\frac{f}{1\,\mathrm{kHz}}\right) - 20\log\left(\frac{r}{0.1}\right)$$

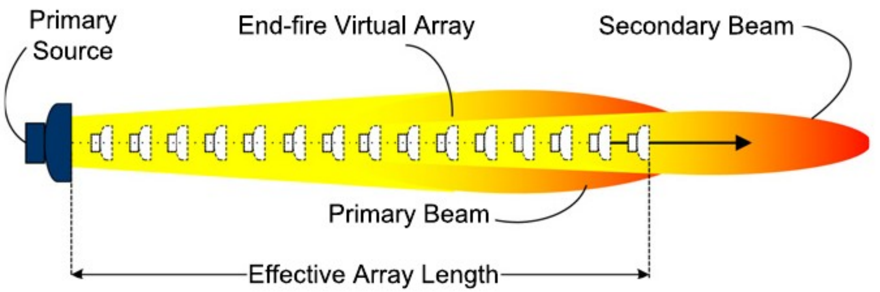
  - Strongly frequency dependent (constant displacement source ≠ constant acceleration source)
  - 100 Hz: 21 dB SPL @ 10 cm (unaudible / hardly audible)

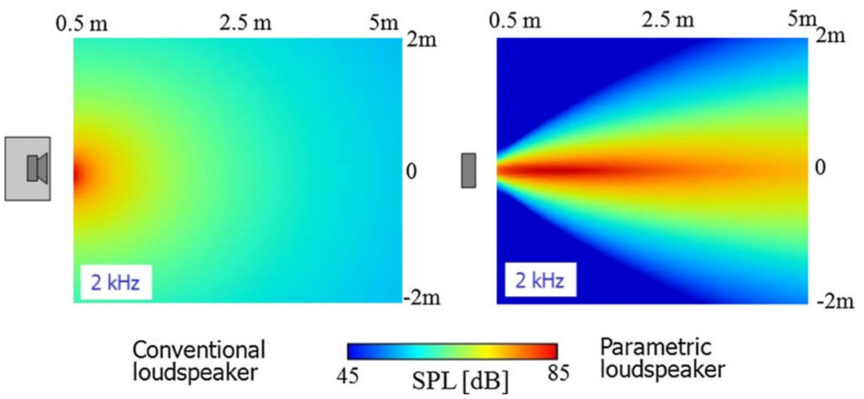
Frequency	$L_{p,\mathrm{on-axis}}$ 10 cm distance	L_w	θ_{FWHM}
100	21 dB SPL	6 dB SWL	/
1000	61 dB SPL	46 dB SWL	/
10000	101 dB SPL	85 dB SWL	/
20000	113 dB SPL	95 dB SWL	/
40000	125 dB SPL	100 dB SWL	32°
80000	137 dB SPL	106 dB SWL	15°

$$\hat{p}(r,\omega,\theta) = \hat{v}\frac{iz_0ka^2}{2r}\left[\frac{2J_1(ka\sin\theta)}{ka\sin(\theta)}\right]\exp(-ikr)\,,$$

Parametric end-fire array

- The theory has been developed mainly in de “60s (Westervelt) with underwater acoustics applications in mind.
- An amplitude-modulated carrier frequency wave generates sound at low frequencies.
- Our research question is: for a small source, is parametric driving a viable solution, compared to “normal” audible driving?





Source: Gan et al: a review of parametric acoustic array in air, 2012

Parametric array - theory

- Suppose we have a quadratic distortion. Some trigonometry

$$\sin^2(\omega t) = \underbrace{\frac{1}{2}}_{\text{DC, streaming,thermoacoustics}} - \underbrace{\frac{1}{2}\cos(2\omega t)}_{\text{Harmonic distortion}}$$

$$(\sin(\omega_1 t) + \sin(\omega_2 t))^2 = \underbrace{\frac{1}{2}}_{\text{DC}} + \underbrace{\cos([\omega_1 - \omega_2]\, t)}_{\text{"Useful" IMD, audible?}} - \underbrace{\cos([\omega_1 + \omega_2]\, t) + \mathcal{O}(1)\left[\cos(2\omega_1 t) + \cos(2\omega_2 t)\right]}_{\text{Ultrasonic crap}}$$

Parametric array - theory

- Some trigonometry
  - Modulation of a carrier wave:

$$\left[\left(\frac{1}{2}+\frac{1}{4}\underbrace{\sin(\omega_m t)}_{\text{Audio band modulation signal}}\right)\underbrace{\sin\left(\omega_c t\right)}_{\text{Carrier wave}}\right]^2 = \left[\underbrace{\mathcal{O}(1)}_{\text{DC}}+\underbrace{\mathcal{O}(1)\cos\left(\omega_m t\right)}_{\text{Audible IMD}}+\underbrace{\mathcal{O}(1)\cos\left(2\omega_m t\right)}_{\text{Distorted audible IMD}}\right.$$

$$\left.+\underbrace{\mathcal{O}(1)\cos\left(2\omega_c t\right)+\mathcal{O}(1)\sin\left(2\left(\omega_m-\omega_c\right)t+\phi_1\right)+\ldots\text{some more}}_{\text{Ultrasonic crap}}\right]$$

Theory

- Starting with Westervelt equation:

$$\underbrace{\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)}_{\text{wave operator}} p = - \underbrace{\frac{\delta}{c_0^4}\frac{\partial^3 p}{\partial t^3}}_{\text{thermoviscous attenuation}} + - \underbrace{\frac{\beta}{\rho_0 c_0^4}\frac{\partial^2}{\partial t^2}(p - p_0)^2}_{\text{nonlinearity}},$$

where

$$\delta = \frac{1}{\rho_0}\left[\left(\frac{4}{3}\mu + \zeta(\omega)\right) + \kappa\left(\frac{1}{c_v} - \frac{1}{c_p}\right)\right]$$

where ζ is the frequency-dependent bulk viscosity.

Perturbation analysis

Substitute:

$$p = \varepsilon \underbrace{p_c(\boldsymbol{x}, t)}_{\text{carrier}} + \varepsilon^2 \underbrace{p_s(\boldsymbol{x}, t)}_{\text{scattered wave}} + \mathcal{O}\left(\varepsilon^{>2}\right),$$

The scattered wave has possible frequency components in the audible part.
the perturbation. First order is ultrasonic wave:

$$c_0^2 \nabla^2 p_c - \frac{\partial^2 p_c}{\partial t^2} = -\frac{D}{c_0^2} \frac{\partial^3 p_c}{\partial t^3} + \mathcal{O}(\varepsilon)$$

Assuming a set of monochromatic waves for the first order solution:

$$p_c = \Re\left[\sum_{n=0}^{N-1} \hat{p}_{c,i} \exp\left(i\omega_n t\right)\right],$$

$$\underbrace{\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)p}_{\text{wave operator}} = - \underbrace{\frac{\delta}{c_0^4}\frac{\partial^3 p}{\partial t^3}}_{\text{thermoviscous attenuation}} + \underbrace{- \frac{\beta}{\rho_0 c_0^4}\frac{\partial^2}{\partial t^2}\left(p - p_0\right)^2}_{\text{nonlinearity}},$$

Second order model – perturbation analysis

we can transform the first order time domain solution to a set of uncoupled damped Helmholtz equations:

$$\nabla^2 \hat{p}_{c,n} + k_n^2 \hat{p}_{c,n} = \frac{i\omega_i^3 \delta}{c_0^4} \hat{p}_{c,n}.$$

For a 1D wave, a solution ansatz is:

$$\hat{p}_{c,n} = \exp\left(-i\tilde{k}x\right),$$

solving for $\tilde{k}$ yields:

$$\tilde{k} = k_n - i\alpha,$$

where

$$\alpha \sim \frac{\omega_i^2 \delta}{2c_0^3},$$

which is valid as long as $\alpha \ll k$.

Alpha: attenuation of ultrasound in air.

Second order model – audible source term

Solving the Westervelt equation up to second order yields, after subtracting the first order result:

$$\left(\nabla^2 - \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}\right)\left(\varepsilon^2 p_s\right) = -\varepsilon^2 \frac{\delta}{c_0^4}\frac{\partial^3 p_s}{\partial t^3} - \varepsilon^2 \underbrace{\frac{\beta}{\rho_0 c_0^4}\frac{\partial^2}{\partial t^2}\left(p_c^2\right)}_{q(t)} + \mathcal{O}\left(\varepsilon^3\right)$$

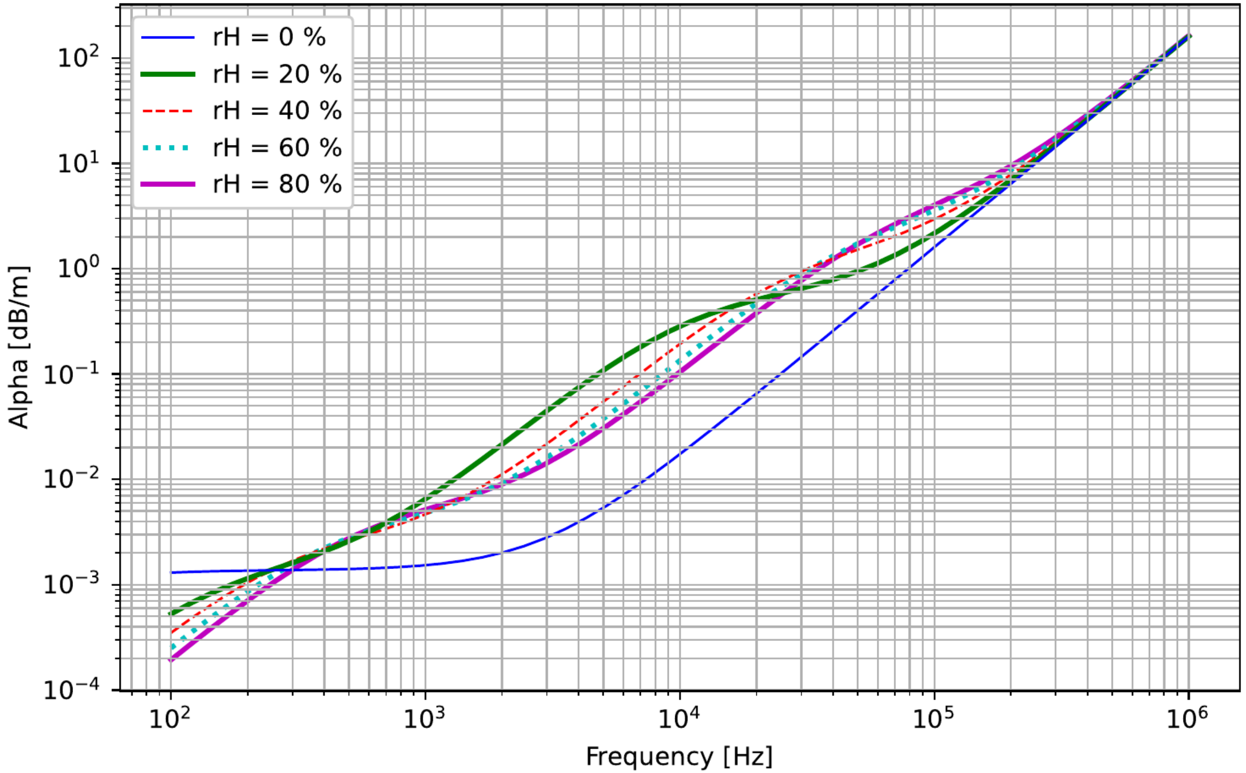
Going to COMSOL notation finally, for the scattered waves, we can write in frequency domain:

$$-\frac{1}{\rho_0}\nabla^2 p_s - k_s^2 p_s = \hat{q} = \frac{\beta}{\rho_0^2 c_0^4}\mathcal{F}\left[\frac{\partial^2 p_c^2}{\partial t^2}\right]_{\omega=\omega_s}.$$

- Hence, the ultrasonic carrier waves end up as a “source term” for the scattered / audible wave. The source term is proportional to the square of the amplitude of the carrier wave(s).

Ultrasound attenuation in air

- Ultrasound damping in air [dB/m] for several values of the relative humidity (rH).



Double-double logarithmic scale for the acoustic attenuation in dB/m, versus frequency.

How to: generate some audio

For a modulated signal of the form

$$x(t) = x_{\max} \frac{1}{2} [1 + E(t)] \cos(\omega_c t) \,,$$

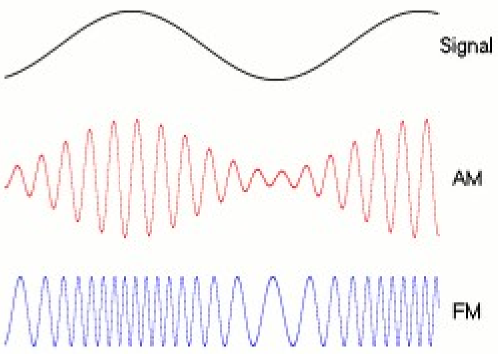
where

$$|E(t)| \leq 1,$$

is the so-called envelope function, the resulting carrier spectrum contains a carrier peak and two side bands, as is known from RF amplitude modulation.

In case the modulation signal is a cosine with radial frequency ω_1:

$$E(t) = \cos(\omega_1 t) \,,$$



How to: generate some audio

- Right: long story.
- Key take-away: an ultrasonic sine wave that is AM-modulated with an audible sine wave generates:
  - An audible wave at the modulation sine wave frequency
  - An audible wave at double the frequency of the modulation sine wave. This is not nice!

after utilizing some trigonometric identities, we can write for the Fourier components of the displacement boundary condition x_c:

$$x = x_{\max}\left[\frac{1}{2}\cos(\omega_c t) + \frac{1}{4}\cos((\omega_1 - \omega_c)t) + \frac{1}{4}\cos((\omega_c + \omega_1)t)\right].$$

From driving the boundary with these displacement frequency components, three acoustic pressure frequencies are generated. These are called the "pump waves". The resulting pump pressure waves are called $p_-(\boldsymbol{x})$,$p_c(\boldsymbol{x})$ and $p_+(\boldsymbol{x})$, and are computed using the linear Helmholtz equation in the domain, for given boundary conditions on the circular piston.

After some mathematical treatment, we find for the interaction in the audible domain the following terms:

$$\begin{aligned} p^2|_{\text{audible}} &= p_+p_-\cos(\alpha_+ - \alpha_- + 2\omega_1 t) \\ &\quad + p_+p_c\cos(\alpha_+ - \alpha_c + \omega_1 t) \\ &\quad + p_-p_c\cos(\alpha_c - \alpha_- + \omega_1 t), \end{aligned}$$

which is in phasor notation:

$$p^2|_{\text{audible}} = \Re\left[\hat{p}_+\hat{p}_-^*\exp(2i\omega_1 t) + (\hat{p}_+\hat{p}_c^* + \hat{p}_c\hat{p}_-^*)\exp(i\omega_1 t)\right].$$

And hence:

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left[p^2|_{\text{audible}}\right] = -\omega_1^2\Re\left[4\hat{p}_+\hat{p}_-^*\exp(2i\omega_1 t) + (\hat{p}_+\hat{p}_c^* + \hat{p}_c\hat{p}_-^*)\exp(i\omega_1 t)\right].$$

Results in $\hat{q}(\omega)$

$$\hat{q} = \frac{\beta}{\rho_0 c_0^4}\mathcal{F}\left[\frac{\partial^2 p_c^2}{\partial t^2}\right]_{\omega=\omega_s}.$$

And hence, we can write for the single frequency audible scattered Helmholtz equation:

$$-\frac{1}{\rho_0}\nabla^2 p_{s,1} - k_{s,\omega_1}^2 p_{s,1} = -\frac{\beta\omega_1^2}{\rho_0^2 c_0^4}(\hat{p}_+\hat{p}_c^* + \hat{p}_c\hat{p}_-^*),$$

and for component at double the envelope frequency:

$$-\frac{1}{\rho_0}\nabla^2 p_{s,2} - k_{s,2\omega_1}^2 p_{s,2} = \frac{4\beta\omega_1^2}{\rho_0^2 c_0^4}\hat{p}_+\hat{p}_-^*$$

How to: generate some audio

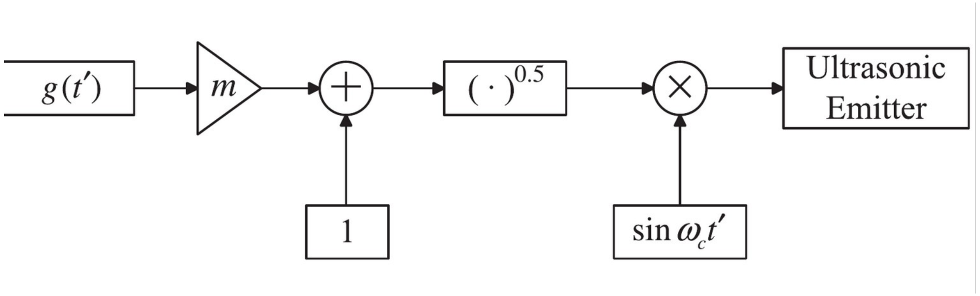
- Solutions:

Berktay solution

Berktay's model: spherically spreading pencil beam. width $2\phi_c$. Within this cone, the intensity is assumed ~ constant. Berktay equation for far-field (at distances much larger than the viscothermal attenuation length α):

$$p(z,t) = \frac{\beta p_c^2 a^2}{4\rho_0 c_0^4 z \alpha_c} \frac{\partial^2}{\partial t^2} \left\{ \left[1 + E\left(t - \frac{r}{c_0} \right) \right]^2 \right\}, \quad (34)$$

- Myriad:
  Square root amplitude modulation: compute the square root of the signal (after offsetting it), and use this as the modulation signal.
  - Others:
    - Double sideband AM
    - Modified AM
    - Single-sideband AM
    - Recursive single sideband AM



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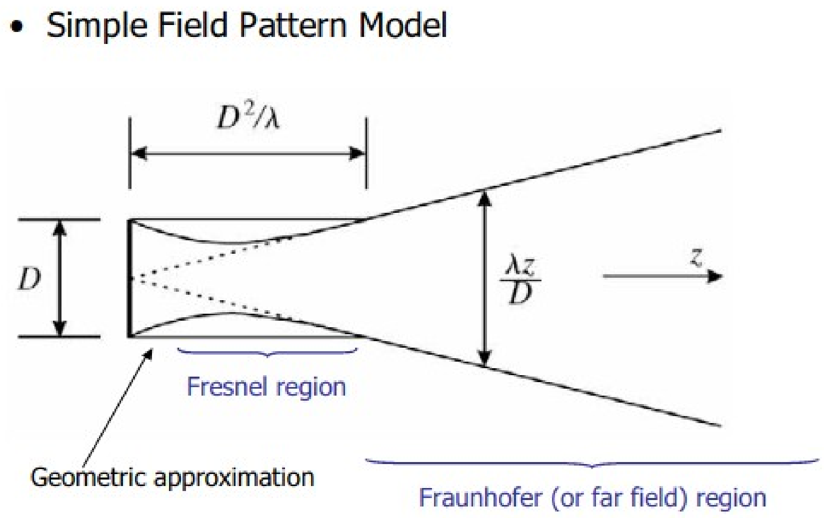
- Small source parametric arrays: simulations

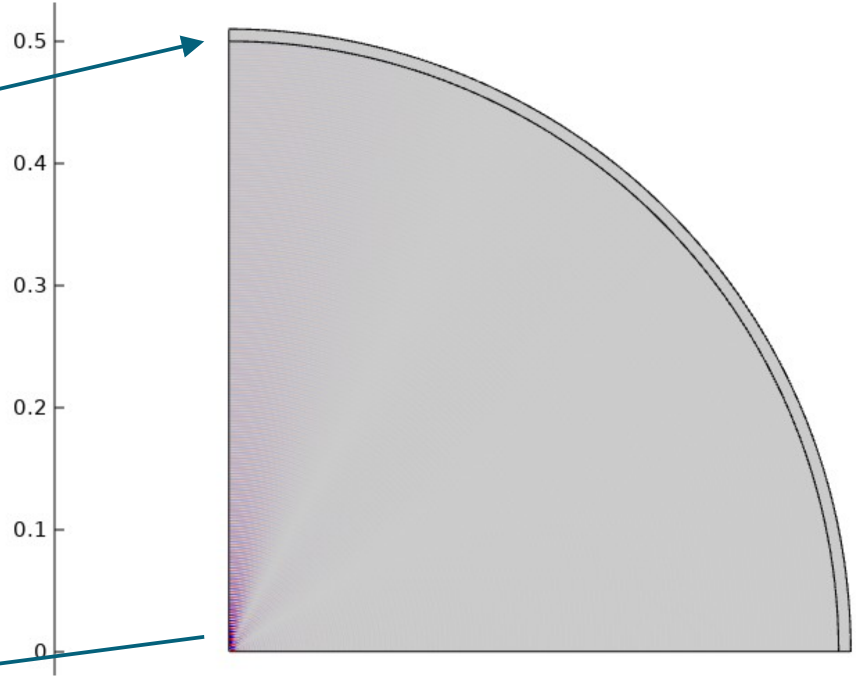
Why all the theory?

- We have a small source:
  - Geometric spreading results in fast decay of ultrasound generation.
  - Near-field effects are more important! Impanct on parametric resonance.

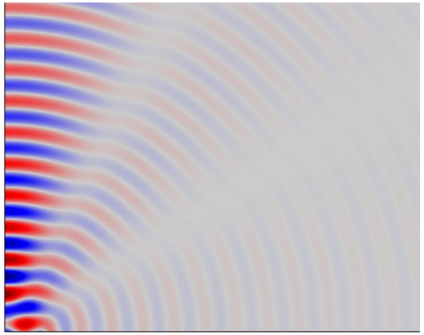


Simulation

- A Comsol model has been implemented to predict the strength of the audible frequency waves due to intermodulation distortion.
  - Axisymmetric model 0.5 x 0.5 m
  - Perfectly matched layer
  - Ultrasonic carrier freq: 80 kHz



Zoom



Expensive simulations

- Wavelength ~ 0.7 mm. Domain 0.5 m. 6 elements / wavelength?
- Solve transformed Helmholtz equation, Slowly varying envelope (SVE):
- Original idea: adjusted from Ysbrand Wijnant’s amplitude-phase split ;).

$$\nabla^2 p + k^2 p = -\rho_0 Q \tag{37}$$

Substitute:

$$p = A(\boldsymbol{x}) \exp\left(-ikR\right), \tag{38}$$

where

$$R = \sqrt{x^2 + y^2 + z^2} \tag{39}$$

Results in the following differential equation for A:

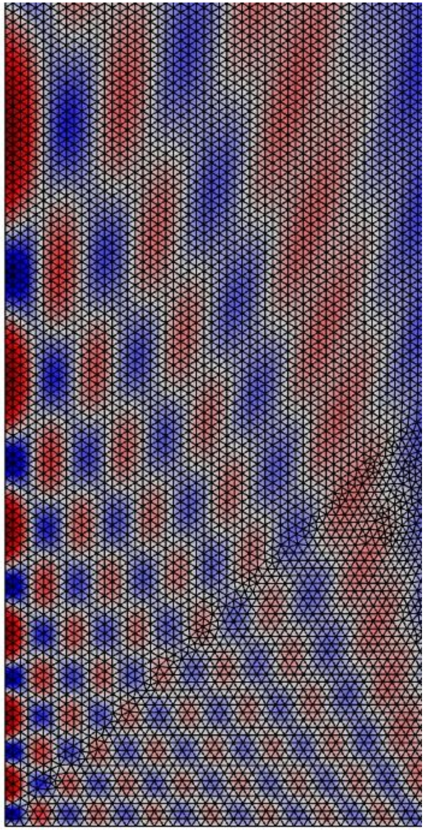
$$\Delta A - 2ik \left[\frac{\partial}{\partial R} + \frac{1}{R}\right] A = -\rho_0 Q \exp\left(ikR\right) \tag{40}$$

Which is in cylindrical coordinates:

$$\underbrace{\frac{\partial^2 A}{\partial z^2} + \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r}}_{\Delta A_{\text{cyl.coords}}} - 2ik \left[\frac{1}{R} + \frac{z}{R}\frac{\partial}{\partial z} + \frac{r}{R}\frac{\partial}{\partial r}\right] A = -\rho_0 Q \exp\left(ikR\right) \tag{41}$$

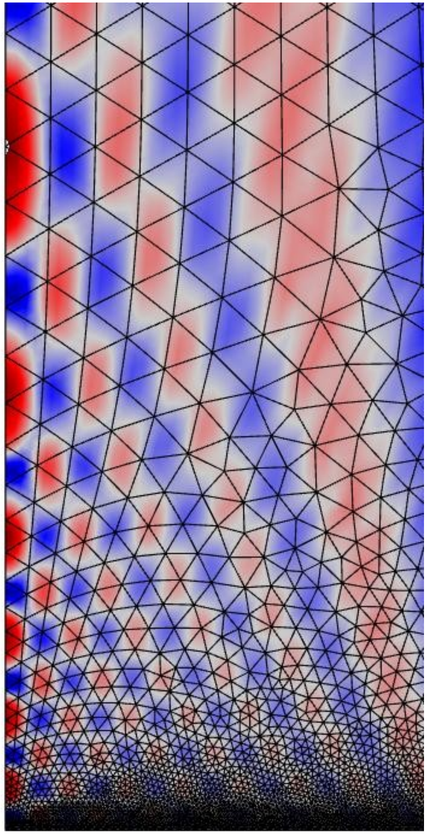
Comparison: normal Helmholtz vs SVE

- Envelope Helmholtz |p|
- SVE Envelope |A|



Sub-wavelength mesh

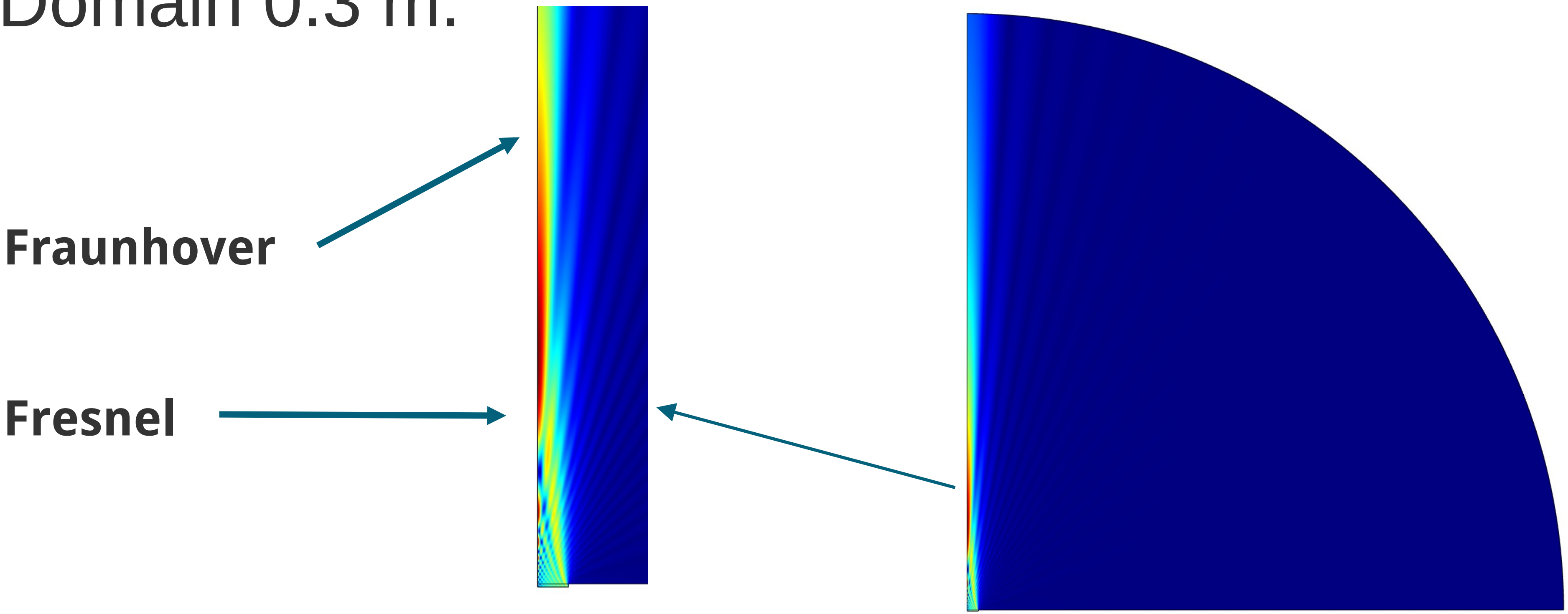
Super-wavelength mesh



SVE: HUGE domain simulations ;)

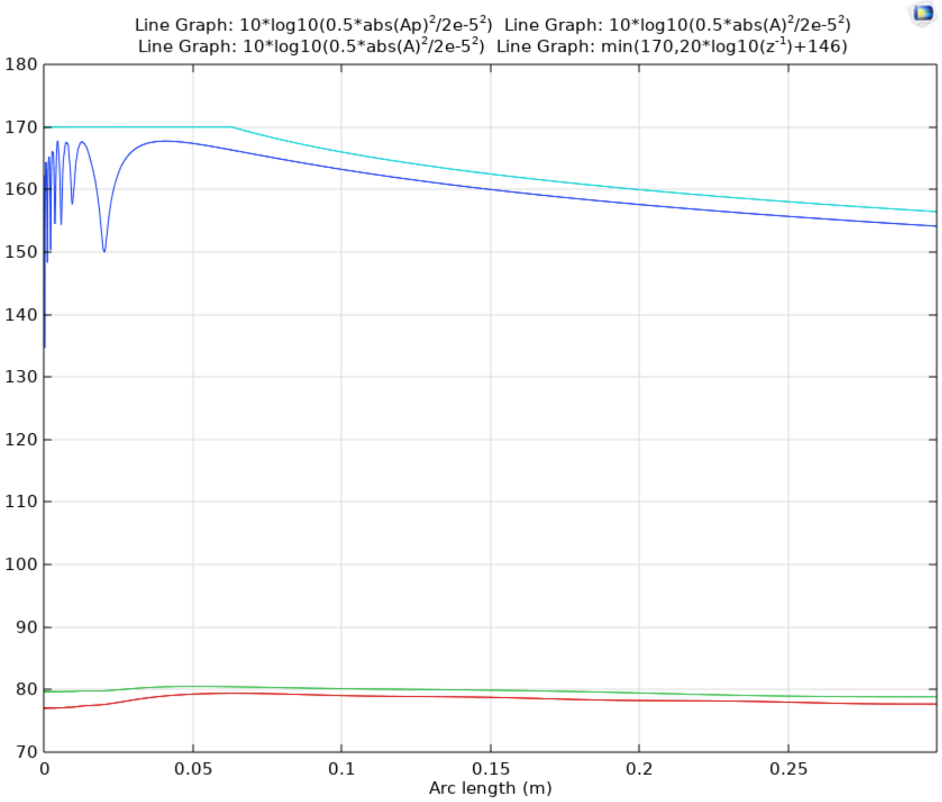
- Pump wave 500 kHz
- Audible wave, 1kHz. Domain 0.3 m.

Figure: SPL of pump wave [dB scale]



Results – on-axis levels

- Turquoise: geometric spreading equation
- Blue: pump wave on-axis SPL
- Green: 2kHz level
- Red: 1 kHz level
- Too low levels based on Berktay prediction

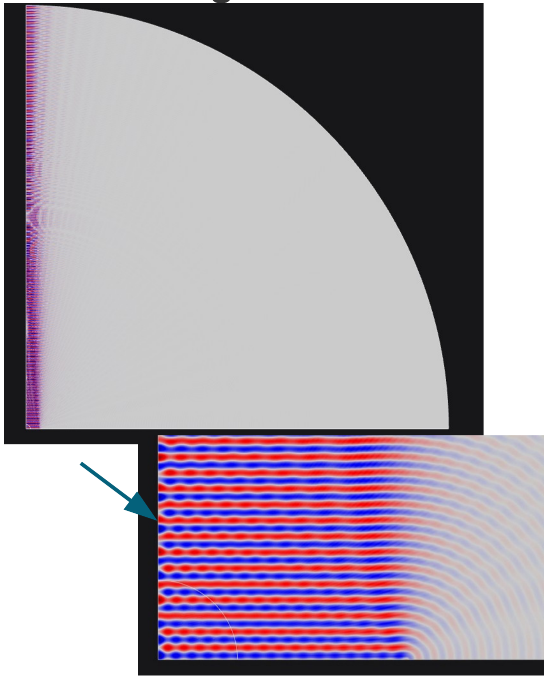


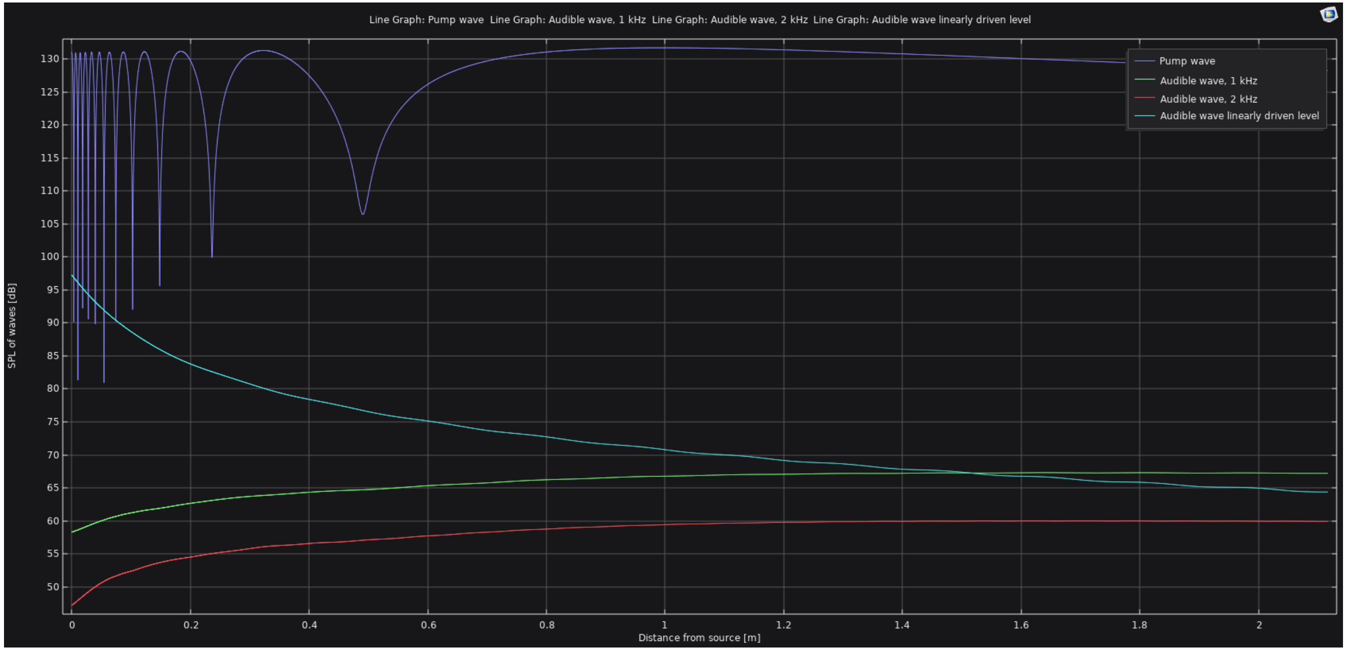
Why not so much audible SPL?

- Berktay prediction is failing
- The on-axis level should be enough
- Large near field region
- Parametric resonance does not occur
- “Diffusion” of the difference wave is the main problem. The created beam is too thin w.r.t. the wavelength of the difference frequency wave.

Not that much output, larger driver may be?

- Radius 12 times as large. Pump wave at ~ 130 dB SPL
- **Larger size = 144 cm²**





Now parametric wins at ~ 130 dB SPL pump wave

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Wrap-up

Conclusions

- We build up understanding and simulation tools to predict the performance of parametric acoustic arrays. Even including the near field effects.
- At higher frequencies: attenuation results in shorter beam
  - Larger Fresnel region, where no parametric effect occurs
- Other idea is to create a long tube where the parametric effect is contained at high pump wave amplitude. But this again generates a strong reflection coefficient at the tube exit, severely limiting the low frequency output. Plus you need a long tube.
- At the scale of a single driver of 1 cm2 it does not seem to be possible to produce a strong audible parametric beam.
  - Scaling of driver size is not beneficial for parametric array effect.
  - Multiple drivers in an array?

Notes

- What you can’t hear, might still hurt you:
  - High levels of ultrasound might cause damage to biological systems.

- Table with limits on acceptable SPL levels:

Table 1

Guidelines for the safe usage of ultrasound that are recommended in various countries. All values in decibels are upper limits for whole-day exposure. Table extracted from [47].

One-third-octave band centre frequency (kHz)	10	12.5	16	20	25	31.5	40	50	63	80	100
Source	Band level (dB)										
IRPA, 1984	–	–	–	75	110	110	110	110	110	110	110
Australia, 1981	75	75	75	110	110	110	110	110	–	–	–
USA, 1981	80	80	80	105	110	115	115	115	–	–	–
Canada, 1980	80	80	80	110	110	110	110	110	–	–	–
Sweden, 1978	–	–	–	105	110	115	115	115	115	115	115
USSR, 1975	–	75	85	110	110	110	110	110	110	110	110
Norway, 1978	–	–	–	–	—120 (octave)—			—120 (octave)—			–

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