Faster Thermoviscous Acoustic Simulations

For Micro-Acoustics

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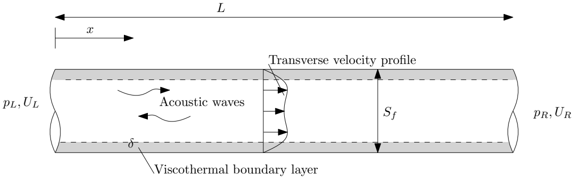
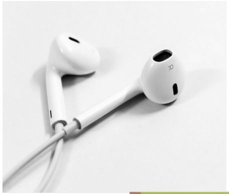
Table of Contents

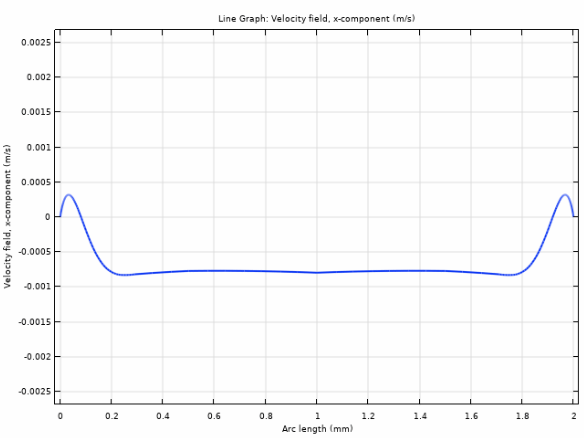
Thermoviscous acoustics





- Many devices related to hearing and sound contain narrow ducts for propagating sound to/from the ear. I.e:
  - Hearing aids
  - Earbuds
  - Hearing protection
- These narrow ducts introduce thermoviscous damping of acoustic waves.
- These effects need to be taken into account in the modeling.
  - COMSOL Thermoviscous Acoustics Physics Interface = numerically expensive
  - Other methods
    - Boundary Layer Impedance ⇒ only for wide geometries
    - Narrow Region Acoustics ⇒ only for prismatic geometries

Thermoviscous acoustics



- Important parameters:
  - Viscous penetration depth: δ_ν
  - Thermal penetration depth: δ_κ

Table of Contents

- Start with, a Newtonian ideal gas, the governing equations are continuity, momentum and the energy equation, closed with the ideal gas equation of state. No external forces, heat sources and negligible gravity.
- Assume small perturbation in time harmonic form. No mean flow, uniform mean temperature and pressure. Then one ends with the Linearized Navier Stokes equations:

$$i\omega\rho + \rho_0 \nabla \cdot \boldsymbol{u} = 0 \quad \Leftarrow \text{continuity,} \tag{1}$$

$$i\omega\rho_0 \boldsymbol{u} = -\nabla p + \nabla \cdot \boldsymbol{\tau} \quad \Leftarrow \text{momentum,} \tag{2}$$

$$\rho_0 c_p i\omega T = i\omega p + \kappa \nabla^2 T \quad \Leftarrow \text{energy,} \tag{3}$$

$$\frac{p}{p_0} = \frac{\rho}{\rho_0} + \frac{T}{T_0} \quad \Leftarrow \text{state,} \tag{4}$$

where:

- $\boldsymbol{u}$: Acoustic velocity phasor [m/s]
- ρ: Acoustic density phasor [kg/m^3]
- i: $\sqrt{-1}$, ω: Radian frequency [rad/s]
- p: Acoustic pressure phasor [Pa]
- c_p: Specific heat at constant pressure [J/kgK]
- T: Acoustic temperature phasor [K]
- κ: Thermal conductivity [W/mK]
- T_0, p_0, ρ_0: Mean values in the field of above described quantities.
- And:

$$\boldsymbol{\tau} = \mu\nabla^2\boldsymbol{u} + (\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u})$$

- Where: μ: dynamic viscosity [Pa·s], λ: Second viscosity [Pa·s].

Further simplifications (1/2)

- To further derive the model, some simplifications are required. The justification of these assumptions is described in detail by R. Kampinga [2, 3, 1]. It is based on a order-of-magnitude analysis and some further assumptions generally applicable in micro-acoustic geometries.
- Apply the following assumptions:
  - The divergence of the velocity field is much smaller than typical single gradients of the velocity field.
  - On the scale of viscothermal boundary gradients, the acoustic pressure is more/less constant.
- Then, we can write for the momentum equation:

$$\boldsymbol{u} = \frac{i}{\omega \rho_0} \left(1 - h_\nu\right) \nabla p, \tag{5}$$

where h_ν satisfies:

$$\nabla^2 h_\nu + \frac{2i}{\delta_\nu^2} h_\nu = 0, \tag{6}$$

and is defined as the "viscous field", where:

$$\delta_\nu^2 = \frac{2\mu}{\rho_0 \omega} \tag{7}$$

- Close to a wall, $h_\nu \to 1$, such that the velocity does not follow the pressure gradient anymore. Far from a wall, $h_\nu \to 0$, such that the inviscid case is retrieved back.

- Apply the following assumption to the energy equation:
  - On the scale of viscothermal boundary gradients, the acoustic pressure is more/less constant.
- Then, we can write for the energy equation:

$$T = \frac{p}{\rho_0 c_p} (1 - h_\kappa) \tag{8}$$

where h_κ satisfies:

$$\nabla^2 h_\kappa + \frac{2i}{\delta_\kappa^2} h_\kappa = 0, \tag{9}$$

and is defined as the "thermal field", where:

$$\delta_\kappa^2 = \frac{2\kappa}{\rho_0 c_p \omega} \tag{10}$$

- Close to a wall, $h_\kappa \to 1$, such that the temperature oscillation goes to 0. Far from a wall, $h_\kappa \to 0$, such that the temperature oscillation responds adiabatically to a pressure oscillation.

Last steps: viscothermal Helmholtz equation derivation

- Eliminate the density by using the equation of state.
- Fill in for the momentum equation in $\boldsymbol{u}$
- Fill in for the temperature the heat equation
- The we find:

$$\nabla \cdot \left(\left(1 - h_\nu \right) \nabla p \right) + k^2 \left(1 + \left(\gamma - 1 \right) h_\kappa \right) p = 0, \tag{11}$$

  where:

$$k = \omega / c_0,$$

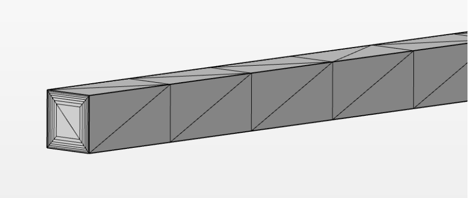
  and

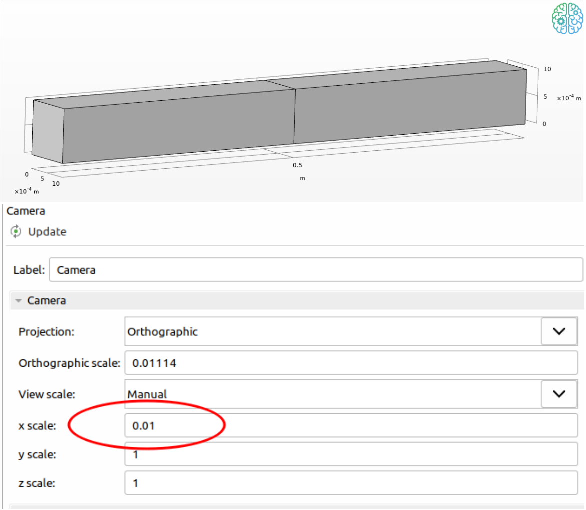
$$\gamma = c_p \left(c_p - R_s \right), \tag{12}$$

  the ratio of specific heats.
- This is an "almost"-Helmholtz equation that can be solved in the same way as the inviscid Helmholtz is already solved in the current "Pressure Acoustics" physics interface.

Table of Contents

- Start with something we know a solution of, before exploring the unknown.
- This is a square duct. Length 1 m, cross section 1x1 mm.
  - These dimensions are such small, that wave damping will lower the amplitude.
- The figure below shows the mesh. Note that boundary layers are required when solving with SLNS.





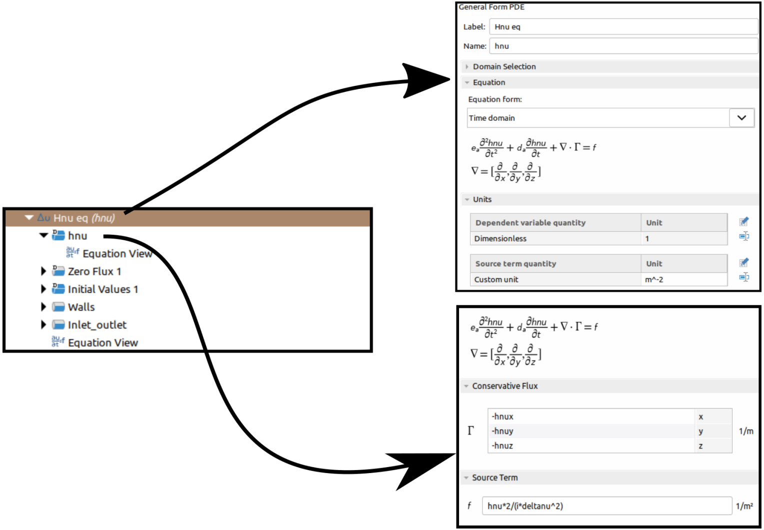
- Solve viscous field equation: Helmholtz equation with a complex "wave number".

$$\nabla^2 h_\nu + \frac{2}{i\delta_\nu^2} h_\nu = 0, \qquad (13)$$

where:

- $h_\nu = 1$ on a no-slip boundary
  - $\nabla \cdot h_\nu \cdot \boldsymbol{n} = 0$ on a symmetry boundary
  - $h_\nu = 0$ on an inlet/outlet, or boundary where the acoustic normal velocity / pressure is prescribed.

- Implemented as "General form PDE"



Model setup - h_κ

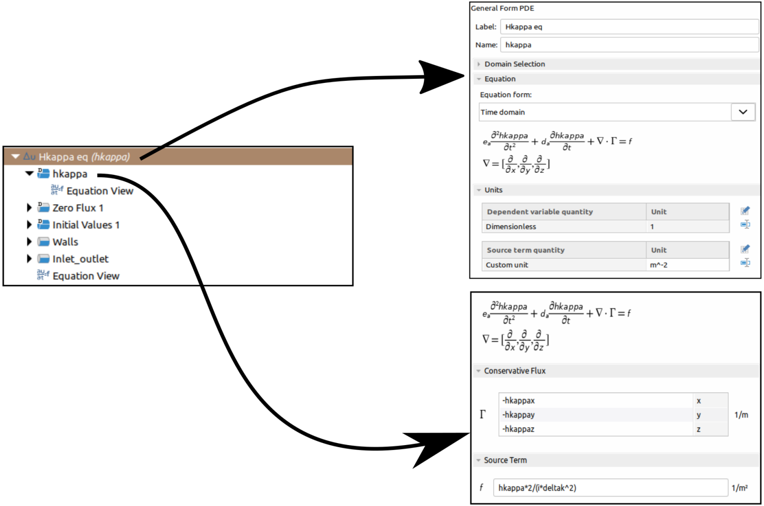
- Solve viscous field equation: Helmholtz equation with a complex "wave number".

$$\nabla^2 h_\kappa + \frac{2}{i\delta_\kappa^2} h_\kappa = 0, \qquad (14)$$

where:

- $h_\kappa = 1$ on an isothermal boundary
  - $\nabla \cdot h_\kappa \cdot \boldsymbol{n} = 0$ on a symmetry boundary
  - $h_\kappa = 0$ on an inlet/outlet, or boundary where the acoustic normal velocity / pressure is prescribed.

- Also implemented as "General form PDE"

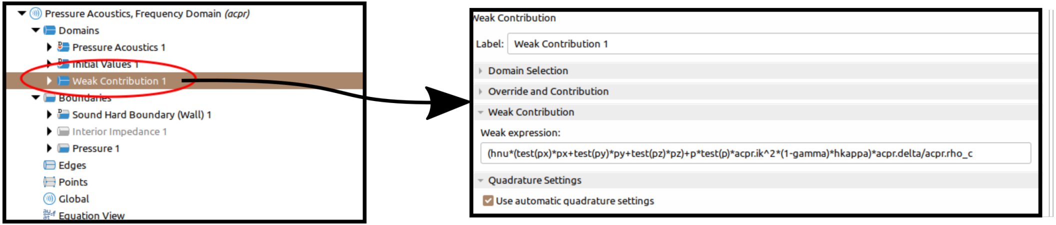


Implementing the adjusted Helmholtz equation for the acoustic pressure

- Adjusted Helmholtz equation incorporating viscous and thermal fields:

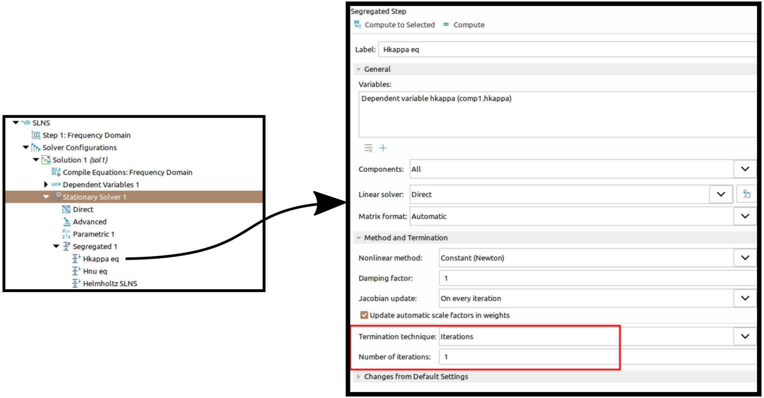
$$\nabla \cdot ((1 - h_\nu)\, \nabla p) + k^2\, (1 + (\gamma - 1)\, h_\kappa)\, p = 0. \tag{15}$$

- We tried to implement this as COMSOL-compatible as possible. So instead of creating a new "General PDE". We tweaked the pressure acoustic interface with a weak contribution.
- It means, splitting up the existing pressure acoustics terms, from the new ones and only adding the new ones.
  - Be careful with signs!



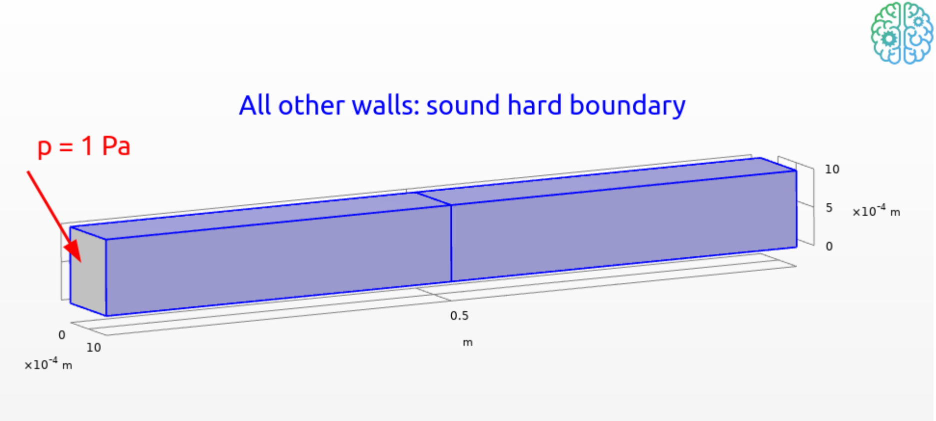
Solver configuration

- To force COMSOL solving this problem sequentially, there are multiple options:
  1. Create separate study steps, in which each of the field is solved, and the use the result of the viscous and termal fields as "Values of variables not solved for".
  2. Replace the default "Fully coupled" solver with a segregated solver. Configure the segregated solver to apply only a single step for all the fields (as it is a sequentially and linear problem).
- Here, we choose option 2.

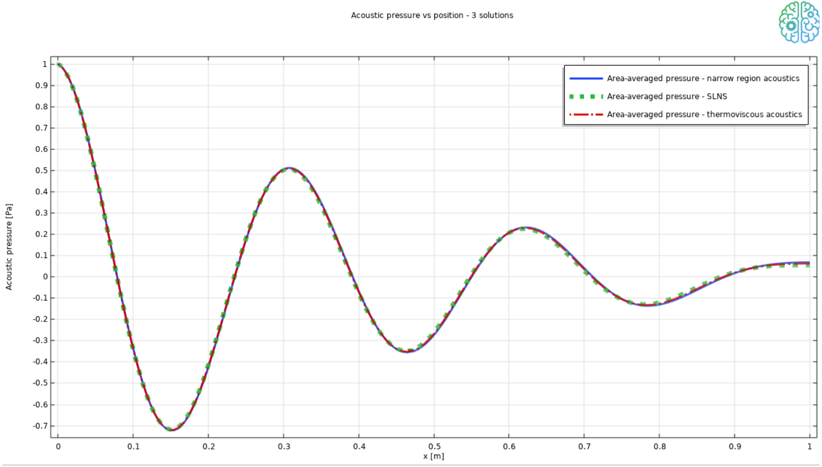


- The first method has as disadvantage that, when performing frequency sweeps, a solution for the viscous and thermal fields needs to be picked correcty, i.e. for frequency x of the Helmholtz solution, also frequency x of the viscous and thermal field solutions should be used. This method is therefore a bit more prone to errors.

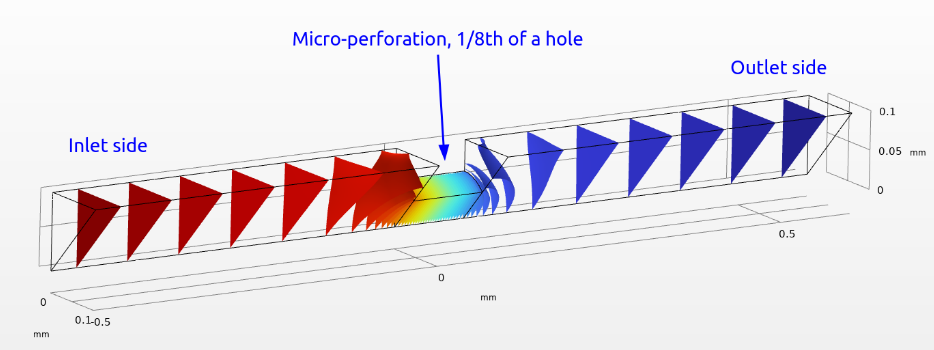
- A simulation has been performed where we put a boundary condition of $p = 1$ Pa at the $x = 0$ inlet, and a "Sound Hard Boundary" on all other sides:



- The figure on the right shows the computed real part of the area-averaged presssure in the duct, for a frequency of 1 kHz.
- As visible, these models predict exactly the same pressure field.
- There is also an analytical solution for this case:
  - $$p(x) = \cos\left(\Gamma\left(L - x\right)\right) \quad (16)$$



- Now we focus on a case were an exact solution is not available: the computation of the equivalent series impedance of a micro-perforate. The figure at the bottom shows a typical "unit cell", of a square pattern micro-perforate with circular holes. We solve this model for 20 frequencies. To reduce the number of DOFS, the symmetry allows to solve for 1/8 of the actual unit cell.
- We show results of wall clock time, as they count for us. There were no other tasks running on the machine.



Benchmark - more advanced case (2/2)

- We use the following machine:
- Intel Xeon Silver 4214, 24 cores, 130 GiB RAM.
- The used solver is Pardiso.
- The number of DOFS is:
  - Pressure acoustics: 20,853 (Quadratic Lagrange)
  - Thermoviscous acoustics: 359,116 (Linear pressure, Quadratic Serendipity T, $\boldsymbol{u}$)
- The table on the right shows the computation times for 20 frequencies.

	Thermoviscous acoustics	SLNS
Wall clock time	16m18s	1m50s
Speedup	1.0	9.2

- There are some other ways of gaining speedups:
  - Frequency-dependent mesh size
  - Decreasing the order of the shape functions (and increasing the mesh size locally where required).

Table of Contents

Conclusions

- In this presentation we showed and derived the Sequentially Linearized Navier Stokes (SLNS) model, and the way it can be implemented in COMSOL.
- We have verified the correct implementation in COMSOL, and have showed that it is able to reach significant speedups for practical computations in micro-acoustics.
- Want to know more?
  - E: j.a.dejong@ascee.nl
  - T: +31 6 189 71 622
- Here are some references:

[1] W. Kampinga. "Viscothermal acoustics using finite elements: analysis tools for engineers." PhD thesis. Enschede, The Netherlands: University of Twente, 2010.

[2] W. Kampinga, Y. Wijnant, and A. de Boer. "An Efficient Finite Element Model for Viscothermal Acoustics." In: *Acta Acustica united with Acustica* 97.4 (2011), pp. 618–631.

[3] W. Kampinga, Y. Wijnant, and A. de Boer. "Performance of Several Viscothermal Acoustic Finite Elements." In: *Acta Acustica united with Acustica* 96.1 (2010), pp. 115–124.